Geometry and Measurement

Most products are packaged
in boxes or cans.
How is a package made?
How do you think the manufacturer
chooses the shape and style of
package? What things need to
be considered?

Look at the packages on this page.
Choose one package.
Why do you think the manufacturer
chose that form of packaging?

What You'll Learn

- Recognize different views of an object.
- Sketch different views of an object.
- Sketch an object.
- Build an object from a net.
- Develop and use a formula for the surface area of a rectangular prism.
- Develop and use a formula for the volume of a rectangular prism.

Why It's Important

- Drawing a picture is one way to help solve a problem or explain a solution.
- Calculating the volume and surface area of a prism is an extension of the measuring you did in earlier grades.

Key Words

- polyhedron (polyhedra)
- prism
- pyramid
- regular polyhedron
- cube
- tetrahedron
- isometric
- pictorial diagram
- icosahedron
- octahedron
- dodecahedron
- frustum
- variable
- surface area

Identifying Polyhedra

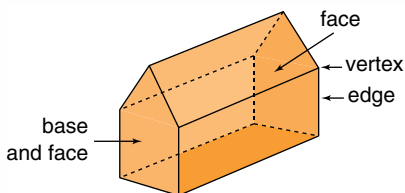A **polyhedron** is a solid with faces that are polygons.
Two faces meet at an edge.
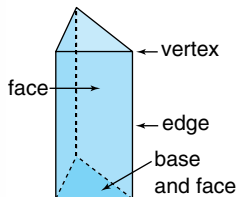Three or more edges meet at a vertex.

A **prism** has 2 congruent bases, and is named for its bases.
Its other faces are rectangles.
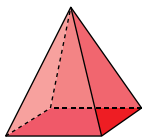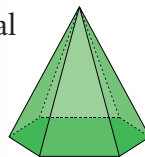
A pentagonal prism

A triangular prism

A **pyramid** has 1 base and is named for that base.
Its other faces are triangles.

A square pyramid
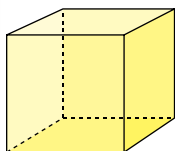
A hexagonal pyramid

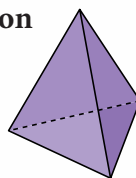A **regular polyhedron** has all faces congruent.
The same number of faces meet at each vertex.
The same number of edges meet at each vertex.

A **cube** is a regular rectangular prism.
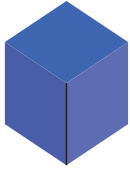
A **tetrahedron** is a regular triangular pyramid.

✓ Check

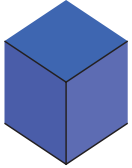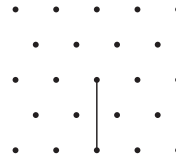Use the pictures of the solids above. Use the solids if you have them.
1. a) How are the solids alike? How are they different?
   b) Name a real-life object that has the shape of each solid.
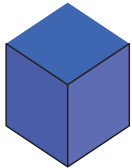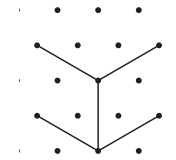
Using Isometric Dot Paper to Draw a Cube

To draw this cube:

Join 2 dots for one vertical edge.

Join pairs of dots diagonally for the front horizontal edges, top and bottom.

Join the dots for the other 2 vertical edges.
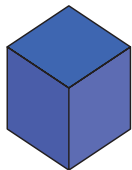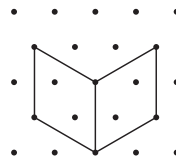
Complete the cube.
Join dots diagonally for the back horizontal edges at the top.
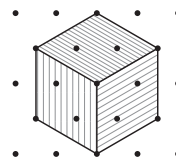
Shade visible faces differently to get a three-dimensional (3-D) look.

Isometric means "equal measure." On isometric dot paper, the line segments joining 2 adjacent dots in any direction are equal.

✓ Check

2. Use isometric dot paper. Draw each object. Use linking cubes when they help.

a)

b)

c)
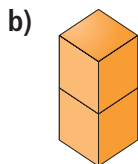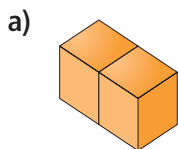
Sketching Views of Solids

Which objects do you see in this picture?

Choose an object. What does it look like from the top? From the side? From the back?

Explore

Work on your own.
Choose a classroom object.

Sketch the object from every view possible. Label each view.
Use dot paper or grid paper if it helps.
Describe each sketch.

Reflect & Share

Trade sketches with a classmate.
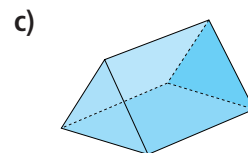Try to identify the object your classmate drew.

Connect

We can use square dot paper to draw each view of the object at the left.
We ignore the holes in each face.

Left side view
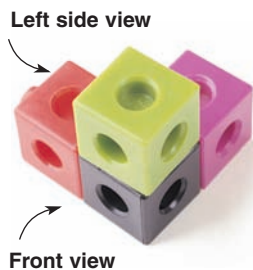
Front view

Back view
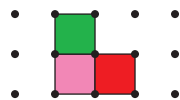
Front view

Left side view
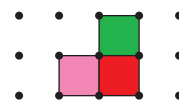
Right side view
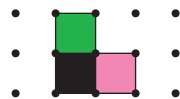
Top view

A triangular prism fits on top of a rectangular prism. The bases of the triangular prism are right isosceles triangles. The rectangular prism has two square faces. Sketch the front, back, side, and top views.
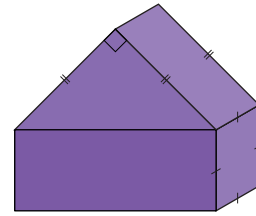
A right isosceles triangle has a 90° angle and two equal sides.

Solution

The front view and back view are the same. Sketch a right isosceles triangle on top of a rectangle.
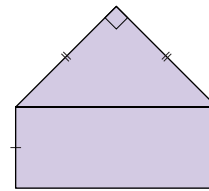
Front view

Side view

The two side views are the same. The side face of the triangular prism is a rectangle. Sketch a rectangle on top of a square.
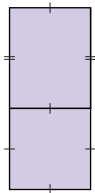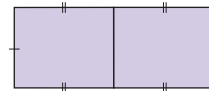
Top view

The top view is two congruent rectangles.

Practice

Use linking cubes when they help.

1. Many signs are views of objects. Identify the view (front, back, side, or top) and the object on each sign.

a) Child care   **b)** Airport transportation   **c)** Fully accessible

2. Use linking cubes. Make each object A to E. Figures J to Q are views of objects A to E. Match each view (J to Q) to each object (A to E), in as many ways as you can.

A

B

C

D

E

J

K

L

M

N

P

Q
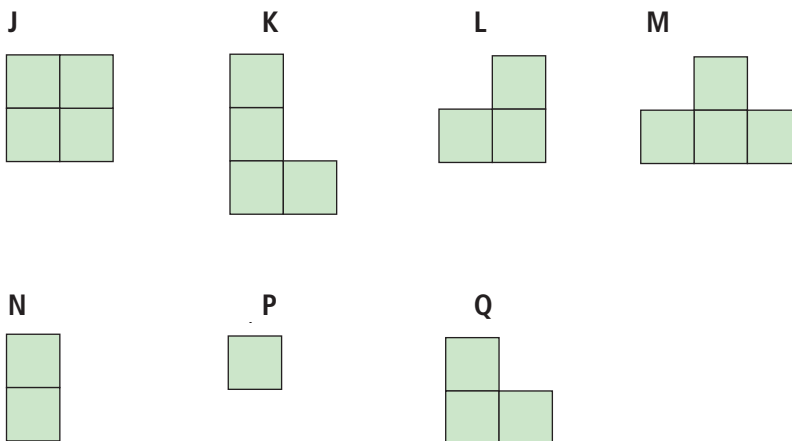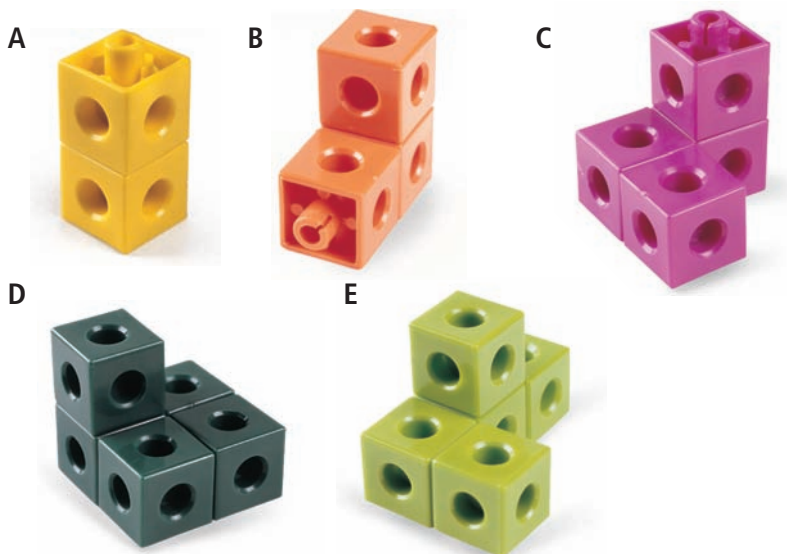
3. Use linking cubes. Make each object. Use square dot paper. Draw the front, back, side, and top views of each object.

a)

b)

c)

14 cm

12 cm

4 cm

12 cm

20 cm

12 cm
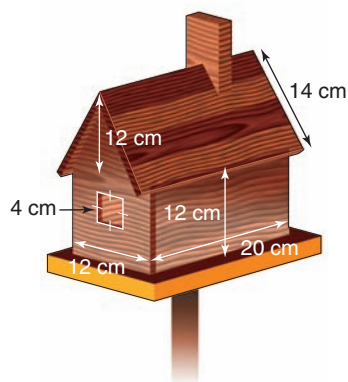
4. Sketch the top, front, side, and back views of the birdhouse at the left. Label each view.

5. Design a road safety or information sign for each situation. Tell which view you used. Explain how each sign shows the information.
  a) playground ahead   **b)** tennis court
  c) skateboarding allowed   **d)** no flash cameras allowed

6. Find each object in the classroom. Sketch the front, back, top, and side views of each object.
  a) filing cabinet   **b)** vase   **c)** teacher's desk

7. **Assessment Focus** Use 4 linking cubes. Make as many different objects as possible. Draw the front, back, top, and side views for each object. Label each view. Use your sketches to explain how you know all the objects you made are different.

8. All 5 views of a cube are the same. Use the solids in the classroom.
  a) Are there any other prisms with all views the same? Explain.
  b) Which solid has 4 views the same?
  c) Which solid has only 3 views the same?
  d) Which solid has no views the same?
  If you cannot name a solid for parts b to d, use linking cubes to make a solid.

9. Here is one view of an object. Sketch a possible different view of the object.
  a)   b)   c)
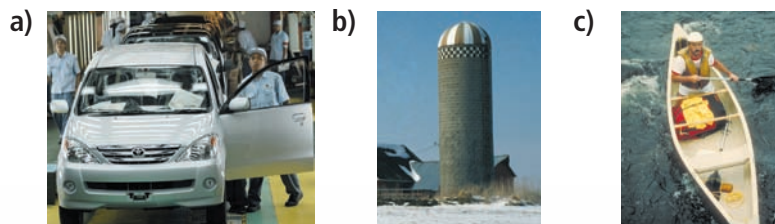
Mental Math

How many:
- days in a year?
- months in a year?
- days in a week?
- hours in a day?
- minutes in an hour?
- seconds in a minute?

Reflect

Choose an object. How many views do you need to sketch, so someone else can identify it? Explain. Sketch the views you describe.

Using a Computer to Draw Views of Solids

Software, such as *The Geometer's Sketchpad*, can be used to draw different views of solids.

Follow these steps.
1. Open *The Geometer's Sketchpad*.

To make a "dot paper" screen:

2. From the **Edit** menu, choose **Preferences**.
Select the Units tab. Check the Distance units are cm.
Click **OK**.

3. From the **Graph** menu, choose **Define Coordinate System**.
Click on a point where the grid lines intersect.
The grid is now highlighted in pink.

4. From the **Display** menu, choose **Line Width**, then **Dotted**.
There are dots at the grid intersections.
Click anywhere on the screen other than the dots.
The dots are no longer highlighted. Hold down the shift key.
Click on each numbered axis and the two red dots.
Release the shift key.
The axes and the dots are now highlighted.

5. From the **Display** menu, choose **Hide Objects**.
The axes and dots disappear.
The screen now appears like a piece of dot paper.
From the **Graph** menu, choose **Snap Points**.

Make this object with linking cubes.
Follow the steps below to create
views of this object.
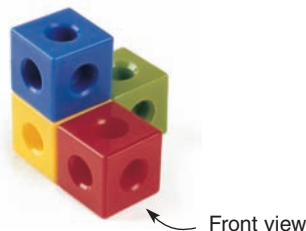
Front view

6. From the **Toolbox** menu, choose **A** (Text Tool).
Move the cursor to the screen and a finger appears.
Click and drag to make a box at the top left.
Inside the box, type: Front View

7. From the **Toolbox** menu, choose (Straightedge Tool).

Move to a dot on the screen below the title.
Click and drag to draw a line segment.
Release the mouse button.
Continue to draw line segments to draw
the front view.
Click to select each line segment.
From the **Display** menu, choose **Line Width**, then **Dashed**.
This draws the line segments as broken lines.
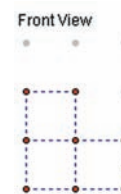
8. From the **Toolbox** menu, click on (Selection Arrow Tool).

Click to select the four corners of the top square,
in clockwise order.
From the **Construct** menu, choose **Quadrilateral Interior**.
From the **Display** menu, choose **Color**, then choose blue.
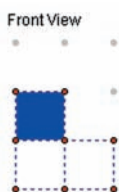
9. Repeat *Step 8* to make
the bottom squares
red and green.
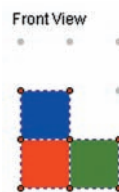
10. Repeat *Step 6* to *9* to draw and label the left side view and
the back view.
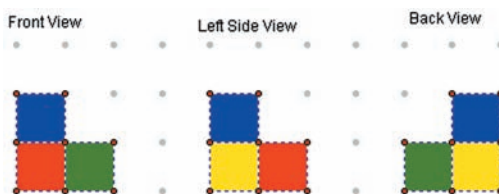
11. Draw the top view and right side view.

✓ **Check**

1. Open a new sketch. Draw different views of objects from
Section 3.1, Practice question 3. Compare the hand-drawn
views with the computer-drawn views.

Explore

Work with a partner.
You will need isometric dot paper and 4 linking cubes.
Make an object so its front, top, and side views are all different.
Draw the object on isometric dot paper so all 4 cubes are visible.

Reflect & Share

Trade isometric drawings with another pair of classmates.
Make your classmates' object.
Was it easy to make the object? Explain.
Compare objects. If they are different, find out why.

Connect

An object has 3 dimensions: length, height, and width or depth.
A drawing is a picture of an object on paper.
It has 2 dimensions: length and width.

➤ We use isometric dot paper to show the 3 dimensions of an object. Parallel edges on an object are drawn as parallel line segments. An object made with 5 linking cubes is shown at the left.

When we draw the object from the front or the top, we see only 4 cubes.

Front view   Top view

Place the object so that all 5 cubes are seen.

Draw the edges in this order:
- vertical edges (red)
- horizontal edges that appear to go down to the right (blue)
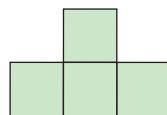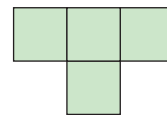- horizontal edges that appear to go up to the right (green)

Shade faces to produce a 3-D effect.
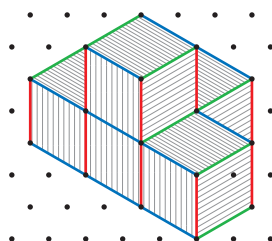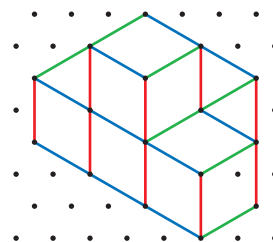
Number Strategies

Simplify.
Use order of operations.
- $17 - 7 \times 2 + 3$
- $15 - 6 \div 2$
- $14 \div 2 \times 3$
- $8 \times 2 + 21 - 6$
- $15 + 3 \times 5$

➢ We can use translations to draw a **pictorial diagram**.
A pictorial diagram shows the shape of an object in 2 dimensions.
It gives the impression of 3 dimensions.
To sketch the rectangular prism above right:

3 cm

2 cm

4 cm
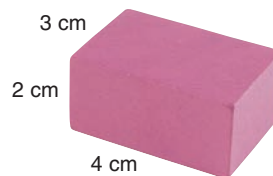
Draw a rectangle 4 cm by 2 cm.

Draw another rectangle that is the image of the first rectangle after a translation up and to the right.
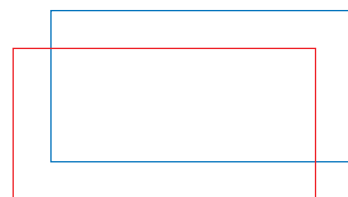
Join corresponding vertices for a sketch of a rectangular prism. Label the dimensions.

4 cm

3 cm

2 cm
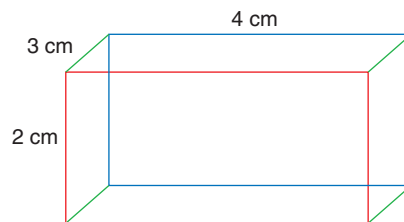
Note that the depth of the prism is 3 cm but, on the drawing, this distance is less than 3 cm. In a pictorial drawing, the depth of an object is drawn to a smaller scale than the length and width. This gives the appearance of 3 dimensions.

We can use similar ideas to sketch a 3-D picture of any object.

Example

Sketch this mug.

Solution

The mug is an open cylinder
with a handle.
Draw an oval for the circular top.
Draw vertical lines for the
curved surface (blue).
Draw half an oval for the
circular base (red).
Draw 2 overlapping curves
for the handle (green).

In a pictorial drawing, a circle is drawn as an oval.
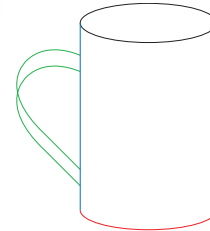
Practice

Math Link

Art

When you view an IMAX film, you "see" it in 3-D. The scenes are filmed from two slightly different angles. This imitates how our eyes view the world. Both films are projected onto the screen to give you the feeling of depth.

Use linking cubes when they help.

1. Make each object. Draw it on isometric dot paper.

a)

b)
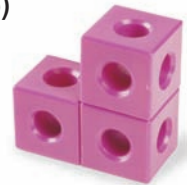
2. Turn each object in question 1.
Draw it a different way on isometric dot paper.

3. Use linking cubes. Make each object.
Draw it on plain paper or isometric dot paper.

a)

b)

4. Here are different views of an object made with linking cubes.

Front   Back   Top   Left Side   Right Side

Make the object.
Draw it on isometric dot paper or plain paper.

5. Sketch a 3-D picture of each object on plain paper.
a)   **b)**   **c)**

6. Square pieces of shelving snap together to make cube-shaped
stacking shelves. All the faces are square and measure
30 cm by 30 cm. A side face costs $1.50.
A top or bottom face costs $1.30. The back face costs $1.10.
Kate built shelves using 9 side faces, 6 back faces,
and 9 top/bottom faces.
a) Use linking cubes to build a possible design.
b) Draw a picture of your design on isometric dot paper.
c) How much did the shelving cost?
d) Is it possible to have the same number of cubes
but use fewer pieces? Explain.
e) If your answer to part d is yes, what is the new cost of
shelving? Explain.

7. **Assessment Focus**   Use 5 linking cubes. Make a solid.
a) Sketch the solid on isometric dot paper.
b) Sketch a 3-D picture of the solid on plain paper.
c) How is sketching a solid on isometric paper different from
sketching it on plain paper? How are the methods alike?
Use your sketches to explain.

Reflect

How do you draw an object to show its 3 dimensions?
Use words and pictures to explain.

Forensic Graphics Specialist

A forensic investigator collects information from a crime scene to find out who was involved and exactly what happened. The forensic investigator also presents evidence in court as required by law.

The forensic graphics specialist visits the crime scene to take photographs and measurements. She may research to compose technical drawings to help with the investigation. When she prepares these drawings, the graphics specialist pays attention to shading and relative line thickness. It is important that she does not present optical illusions such as an object appearing to be "inside out."

In earlier grades, you designed nets for prisms and pyramids.
Now, you will fold nets to make prisms, pyramids, and other polyhedra.

Explore

Work with a partner.
You will need scissors and tape.
Your teacher will give you large copies of the nets below.
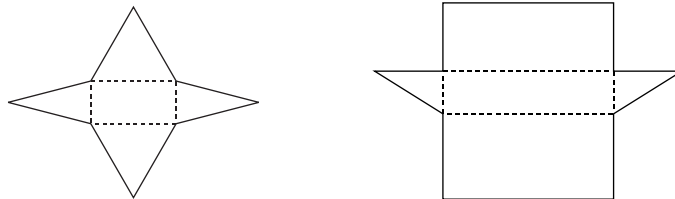
➢ Identify the polyhedron for each net above.
➢ Use a copy of each net.
   Cut it out. Fold, then tape it to make a polyhedron.
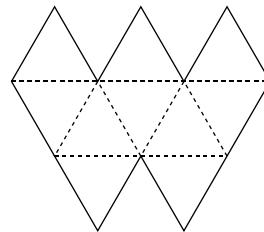➢ Identify congruent faces on each polyhedron.

Reflect & Share

Look at each polyhedron from the top, front, back, and sides.
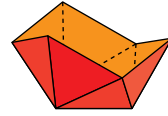Which views are the same? Explain.

Connect

There are 5 regular polyhedra.
You reviewed 2 of them, the cube and the tetrahedron,
in *Skills You'll Need*, page 76.

One regular polyhedron is
an **icosahedron**.
Here is a net for one-half
of an icosahedron.

This net can be cut out and folded.
The net has 10 congruent triangles.

Two of these pieces are taped
together to make an icosahedron.
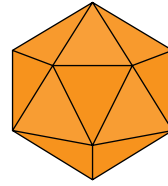
An icosahedron has
20 congruent faces.
Each face is an equilateral triangle.
In the *Practice* questions, you will
make the other 2 regular polyhedra.

A regular **octahedron**
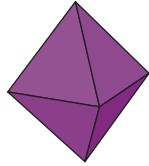
A regular **dodecahedron**
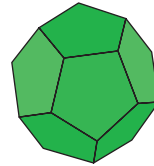
In previous grades, you designed and sketched
nets of prisms and pyramids.
In the *Example* that follows, and in *Practice* questions,
you will investigate nets for other objects.

Example

Fold this net to make
an object.
Describe the object.
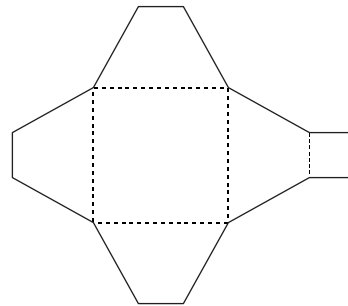
Solution

The net is folded along the broken
line segments. Each edge touches
another edge. The edges are taped.
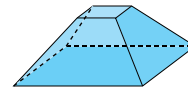
The object looks like the bottom of a pyramid;
that is, a pyramid with the top part removed.
The object is called a **frustum** of a pyramid.

The object has 2 non-congruent square faces, and 4 congruent
trapezoid faces. The 2 square faces are parallel.

Practice

Your teacher will give you a large copy of each net.

1. Fold this net to make a polyhedron.
   a) Identify the polyhedron.
   b) Describe the polyhedron.
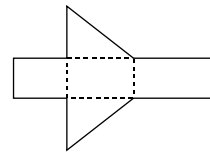
2. Fold this net.
   a) Is the object a polyhedron?
       If so, what are the attributes that
       make it a polyhedron?
   **b)** Identify parallel faces and
       perpendicular faces.
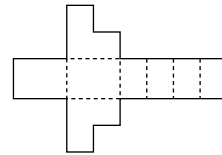
3. Fold two of these nets to make two square
   pyramids with no base.
   Tape the two pyramids together at
   their missing bases.
   You have made a regular octahedron.
   a) Why does it have this name?
   b) Describe the octahedron.
       How do you know it is regular?
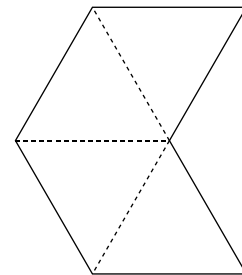
4. Fold each net. Describe each object.
   How are the objects the same? How are they different?
   a)   **b)**
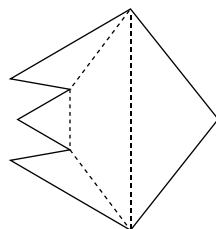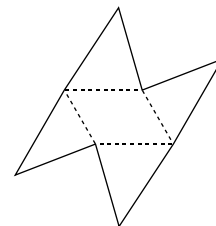
5. Fold two of these nets. Put the open parts together. Tape the pieces to make a regular dodecahedron. Describe the dodecahedron.
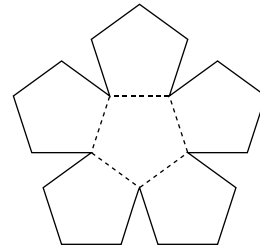
6. A soccer ball is not a sphere. It is a polyhedron. Look at a soccer ball. Explain which polygons are joined to make the ball. How are the polygons joined?

7. **Assessment Focus**   Fold the net shown.
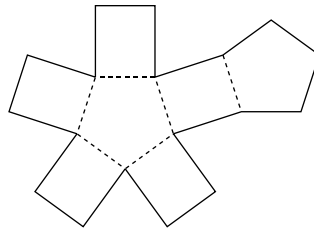
   a) Describe the object formed.
   b) Name the object. Justify your answer.

8. All the nets you have used fold to make objects. Which features must be true for this to happen?

**Take It Further**   **9.** Use isometric dot paper or plain paper. Draw the object that has this net. Explain your thinking.
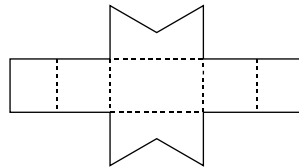
Reflect

Choose a product that has a package in the shape of a polyhedron.
Sketch the package. Include appropriate dimensions.
Cut along the edges to make the net.
Why do you think the manufacturer used this shape for the package?
Can you find a better shape for the packaging? Explain.

Mid-Unit Review

3.1 **1.** Sketch the front, back, top, and side views of each object. Label each view.

a) Television

b) Baseball mitt

c) Chair

3.2 **2.** Make each object. Sketch a 3-D view of the object on isometric dot paper.

a)

b)

3. Sketch a 3-D view of each object on plain paper.

a) **b)**

4. Suppose you had to construct an object from linking cubes. Which would you prefer to use: a drawing of the object on isometric dot paper, or 5 different views on plain paper? Justify your choice. Use diagrams to support your choice.

3.3 **5.** Use a large copy of this net. Fold the net to make an object. Name the object. List its attributes.
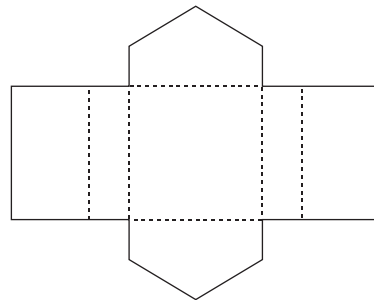
6. Which polyhedra could have each view shown below? Find as many polyhedra as you can for each view.

a) **b)**

Focus Use measurement formulas to introduce the concept of a variable.

Explore

Work with a partner.

➤ Find the area of each rectangle below, in square units.
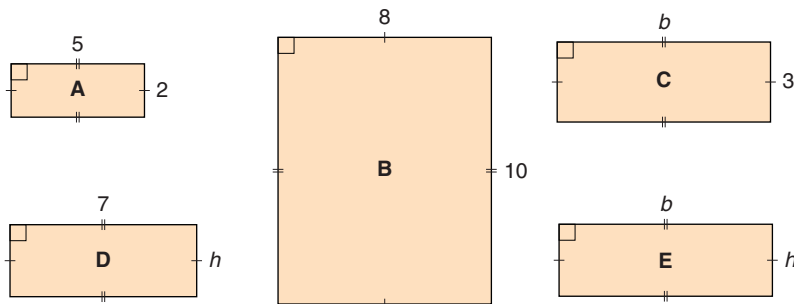
➤ Find the perimeter of each rectangle above, in units.

Reflect & Share

Compare your results with those of another pair of classmates. How did you find the area when you did not know:

- the base?
- the height?
- the base and height?

How did you find the perimeter in each case?

Connect

When we do not know the dimensions of a figure, we use letters to represent them.
For a rectangle, we use b to represent the length of the base, and h to represent the height.

One way to find the area of a rectangle is to use a formula.
The formula for the area of a rectangle is:

$A = \text{base} \times \text{height}$

Then $A = b \times h$

We write $A = b \times h$ as $A = bh$.

The formula for the perimeter of a rectangle is:
$P = 2 \times$ (base + height)
Then $P = 2 \times (b + h)$
We write $P = 2 \times (b + h)$ as $P = 2(b + h)$.

The letters we use to represent the base and height are called **variables**. Variables in a formula can represent different numbers. When we know the values of the variables, we **substitute** for the variables. That is, we replace each variable with a number.

A square is a rectangle with all sides equal.
The formula for the area of a square is:
$A =$ side length $\times$ side length
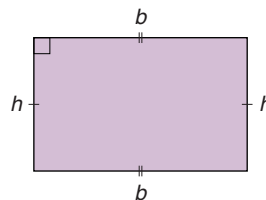Use the variable s for the side length.
Then, $A = s \times s$
   or $A = s^2$
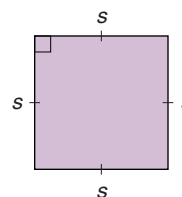
We use exponents to represent repeated multiplication.

The formula for the perimeter of a square is:
   $P = 4 \times$ side length
or $P = 4 \times s$

We write $P = 4 \times s$ as $P = 4s$.

Example

A rectangle has base 15 cm and height 3 cm. Use formulas to find its area and perimeter.
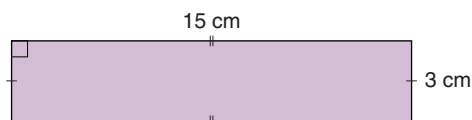
Solution

Area, $A = bh$
$b = 15$ and $h = 3$
Substitute for b and h in the formula.
$A = 15 \times 3$
$A = 45$
The area is 45 cm².

Perimeter, $P = 2(b + h)$
Substitute for b and h.
$P = 2(15 + 3)$ Use order of operations.
$P = 2(18)$ Do the operation in
$P = 36$ brackets first. Then multiply.
The perimeter is 36 cm.
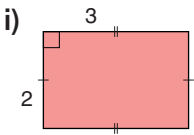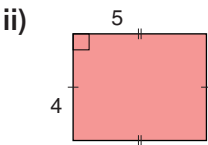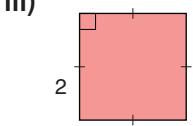
1. Use the formula: $P = 4s$
Find the perimeter of the square with each side length.
a) 5 cm   **b)** 9 cm   **c)** 2 cm   **d)** 8 cm

2. Use the formula for the area of a rectangle: $A = bh$
Find the area of each rectangle.
a) base: 6 cm; height: 3 cm   **b)** base: 11 cm; height: 2 cm

3. a) For each rectangle, what is the value of b and the value of h?

i) 3, 2   **ii)** 5, 4   **iii)** 2

b) Use your answers to part a to explain why
we call b and h variables.
c) What is true about b and h in part a, iii?

4. Find the area and perimeter of each rectangle.
a) base: 12 cm; height: 4 cm   **b)** base: 10.5 cm; height: 3.0 cm

5. Find the perimeter and area of each square.
a) side length: 2.8 cm   **b)** side length: 3.1 cm

6. **Assessment Focus**
Here is another formula for the perimeter of a rectangle:
$P = 2b + 2h$
Write this formula in words.
Explain why there are two formulas for the perimeter
of a rectangle.

Take It Further   **7. a)** How could you use the formula for the perimeter of a
rectangle to get the formula for the perimeter of a square?
b) How could you use the formula for the area of a rectangle to
get the formula for the area of a square?

Reflect

What is a variable?
Why do we use variables to write measurement formulas?

Surface Area of a Rectangular Prism

Focus Use a formula to calculate the surface area of a rectangular prism.

Explore

Work in a group.
You will need several different empty cereal boxes, scissors, and a ruler.

➤ Cut along the edges of a box to make a net.
What is the area of the surface of the box?
➤ Repeat the activity above for 2 other boxes.
➤ Write a formula to find the area of the surface
of a rectangular prism.

Reflect & Share

Compare your formula with that of another group.
Did you write the same formula?
If not, do both formulas work? Explain.

Connect

Here is a rectangular prism, and its net.
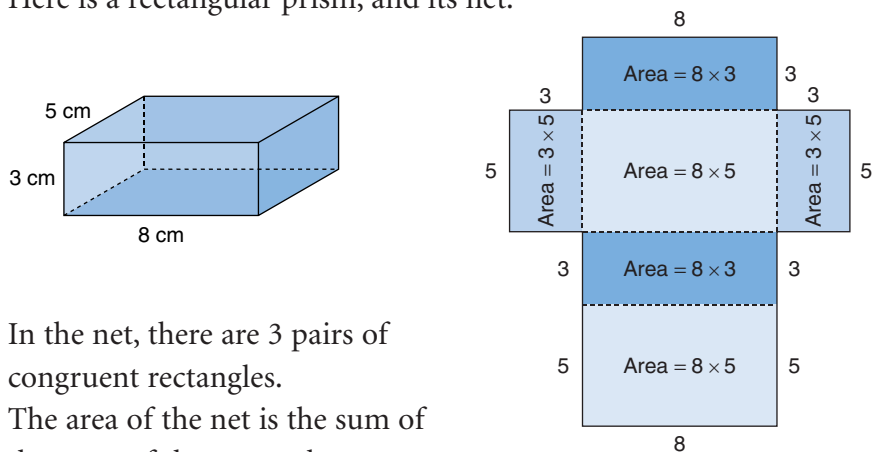
In the net, there are 3 pairs of
congruent rectangles.
The area of the net is the sum of
the areas of the rectangles.
The area of the net = $2(8 \times 3) + 2(8 \times 5) + 2(3 \times 5)$

Use order of operations, with brackets first.

$$= 2(24) + 2(40) + 2(15) \quad \text{Multiply.}$$
$$= 48 + 80 + 30 \qquad\qquad \text{Add.}$$
$$= 158$$

The area of the net is 158 cm².
We say that the **surface area** of the rectangular prism is 158 cm².

We can use the net of a rectangular prism
to write a formula for its surface area.
We use a variable to label each dimension.
The prism has length l, width w, and height h.

In the net, there are 3 pairs of congruent rectangles.
The surface area of the prism is the sum of the
areas of the rectangles.

**Remember that *lw*
means *l* × *w*.**

The surface area of the prism $= 2lh + 2lw + 2hw$
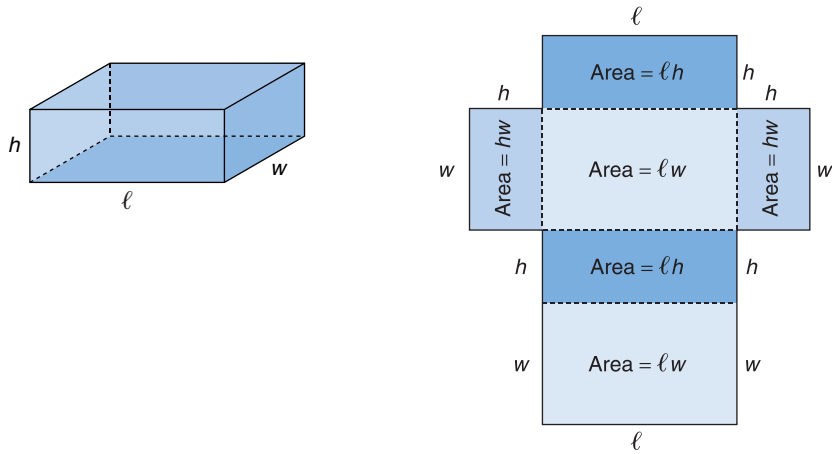We write: $SA = 2lh + 2lw + 2hw$

We can use this formula to find the surface area of a rectangular
prism, without drawing a net first.

Example

Find the surface area of this
rectangular prism.
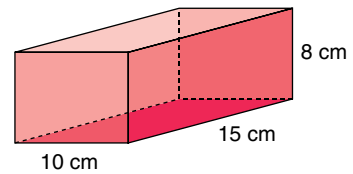
Solution

Use the formula:
$SA = 2lh + 2lw + 2hw$
Substitute: $l = 15$, $h = 8$, and $w = 10$

$SA = 2(15 \times 8) + 2(15 \times 10) + 2(8 \times 10)$
$ = 2(120) + 2(150) + 2(80)$
$ = 240 + 300 + 160$
$ = 700$

The surface area of the rectangular prism is 700 cm^2.

1. Find the surface area of each rectangular prism.

Visualize the net. How does that help to find the surface area?

a) 4 cm 4 cm 8 cm

b) 6 cm 6 cm 6 cm

c) 2 cm 3 cm 7 cm
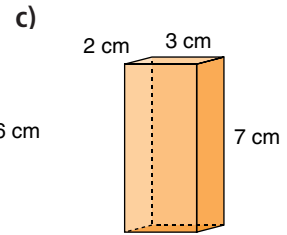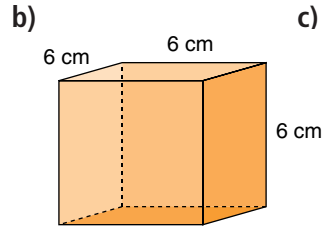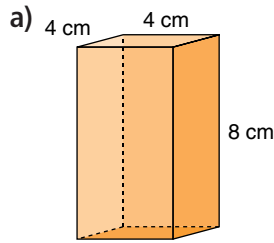
2. Use isometric dot paper or plain paper.
Sketch a rectangular prism with these dimensions:
6 cm by 3 cm by 2 cm.
Find the surface area of the prism.

3. Find the surface area of each rectangular prism.

a) 3.6 cm 5.5 cm 2.0 cm

b) 4.5 cm 7.2 cm 3.0 cm

c) 5.8 cm 5.0 cm 3.5 cm

Mental Math

Estimate each product. What strategies did you use?

Order the products from least to greatest.

• 24.8 × 3.2
• 35.2 × 2.8
• 13.2 × 5.7
• 5.5 × 15.5
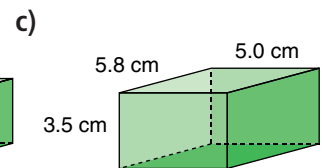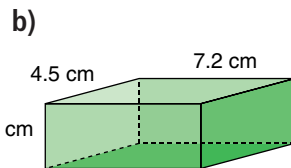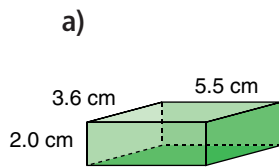
4. Use linking cubes.
a) Find the surface area of a cube with edge length 1 unit.
b) What if the edge length is doubled?
What happens to the surface area?
Make a new cube to find out.
c) What if the edge length is tripled?
What happens to the surface area?
Make a new cube to find out.
d) Predict the surface area of a cube with edge length 4 units.
Explain your prediction. Make a new cube to check.

5. a) Find a rectangular prism in the classroom.
Measure its length, width, and height.
Find its surface area.
b) Suppose each dimension of
the prism is halved.
What happens to the
surface area? Explain.

6. Tanya paints the walls of her family room.
The room measures 3 m by 4 m by 7 m.
The walls need 2 coats of paint. A 4-L can of paint covers 40 m².
  a) How much paint should Tanya buy?
  b) What assumptions do you make? Explain.

7. (Assessment Focus) Sketch a rectangular prism. Label its
dimensions. What do you think happens to the surface area of a
prism when its length is doubled? Its length is halved?
Investigate to find out. Show your thinking.

8. Each object has the shape of a rectangular prism, but one
face or parts of faces are missing. Find each surface area.

a)

10 cm
4 cm   14 cm   7 cm
22 cm

b)

10 cm   10 cm
10 cm   4 cm
4 cm

c)

25 cm
10 cm
18 cm
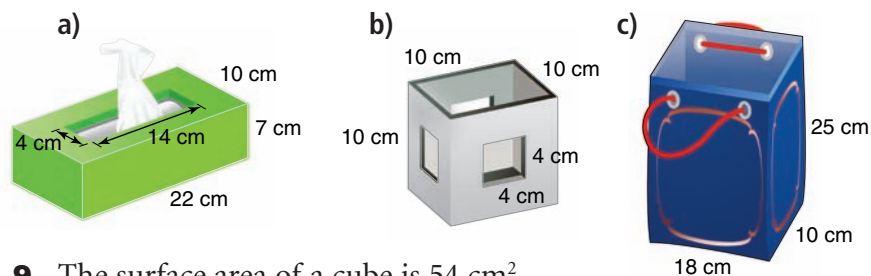
9. The surface area of a cube is 54 cm².
  a) What is the area of one face of the cube?
  b) What is the length of one edge of the cube?

Take It Further

10. A 400-g cereal box measures 20 cm by 7 cm by 31 cm.
A 750-g cereal box measures 24 cm by 9 cm by 33 cm.
  a) Find the surface area of each box.
  b) What is an approximate ratio of surface areas?
    What is an approximate ratio of masses?
  c) Compare the ratios in part b.
    Do you expect the ratios to be equal? Explain.

11. A rectangular prism has a square base with area 4 m².
The surface area of the prism is 48 m².
What are the dimensions of the prism?

12. A rectangular prism has faces with these areas: 12 cm², 24 cm²,
and 18 cm². What are the dimensions of the prism? Explain.

Reflect

Explain how to find the surface area of a rectangular prism
using a formula. Include an example in your explanation.

Volume of a Rectangular Prism

Explore

Work in a group.
You will need several empty cereal boxes and a ruler.

➤ Find the volumes of 3 cereal boxes.
➤ Write a formula you can use to find the volume of a rectangular prism.
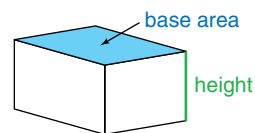➤ Measure a 4th cereal box. Substitute its dimensions in your formula to check that your formula is correct.

Reflect & Share

Compare the formulas for the volume of a rectangular prism and the area of a rectangle. What do you notice? Explain.

Connect

Recall that the volume of a rectangular prism is:
Volume = base area × height

The base of a rectangular prism can be any face of the prism.

The base of the prism is a rectangle.
Label the length l and the width w.
Then, the area of the base is $l \times w$, or lw.
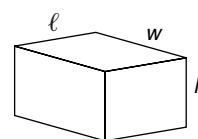
**The height is measured from the base to the opposite face.
The height is perpendicular to the base.**

Label the height h.
Volume = base area × height
$$V = lw \times h$$
$$V = lwh$$
The volume of a rectangular prism is: $V = lwh$

If we let A represent the area of the base, then $A = lw$.

Another way to write the volume is: $V = A \times h$,
  or $V = Ah$

Example

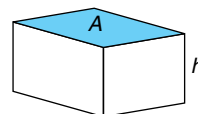A deck of 54 cards fits in a box with dimensions 6.5 cm by 9.0 cm by 1.6 cm. What is the volume of the box? Give the answer to the nearest cubic centimetre.

Solution

Recall that, when the dimensions are measured in centimetres (cm), the volume is measured in cubic centimetres (cm³).

Draw a diagram.
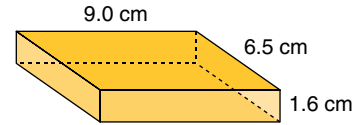The box is a rectangular prism.
Label each dimension.
Use the formula: $V = lwh$
Substitute: $l = 9.0$, $w = 6.5$, and $h = 1.6$
$V = 9.0 \times 6.5 \times 1.6$ Use a calculator.
   $= 93.6$
The volume is 93.6 cm³, or about 94 cm³.

Practice

Use a calculator when it helps.

1. For each rectangular prism: find the area of its base, then find its volume.

a) 8 cm, 5 cm, 3 cm

b) 9 cm, 9 cm, 9 cm

c) 10 cm, 20 cm, 30 cm
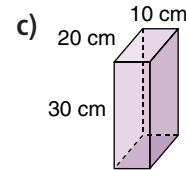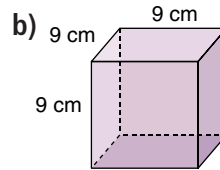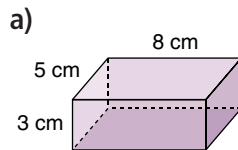
2. Find the volume of each rectangular prism.

a) 3.0 cm, 5.0 cm, 4.5 cm

b) 7.5 cm, 3.2 cm, 4.0 cm

c) 3.5 cm, 2.4 cm, 3.0 cm
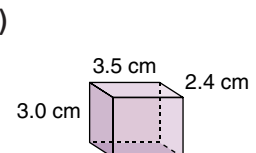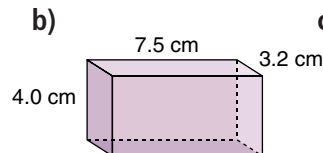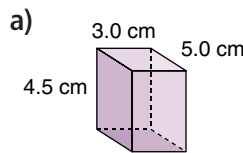
3. Use linking cubes.
Make all possible rectangular prisms with volume 36 units³.
Sketch each prism you make.
Label each prism with its dimensions in units.
How do you know you have found all possible prisms?

4. Philip made fudge that filled a 20-cm by 21-cm by 3-cm pan.
   a) What is the volume of the fudge?
   b) Philip shares the fudge with his classmates. There are 30 people in the class. How much fudge will each person get?
   c) How should Philip cut the fudge so each person gets the same size piece? Sketch the cuts he should make.
   d) What are the dimensions of each piece of fudge?

5. Sketch a rectangular prism. Label its dimensions.
   What do you think happens to the volume of the prism when:
   a) its length is doubled?
   b) its length and width are doubled?
   c) its length, width, and height are doubled?
   Investigate to find out. Show your work.
   Will the results be true for all rectangular prisms?
   How do you know?

6. How can you double the volume of a rectangular prism?
   Does its surface area double, too? Explain.

7. **Assessment Focus**   Use linking cubes.
   a) How many rectangular prisms can you make with 2 cubes? 3 cubes? 4 cubes? 5 cubes? 6 cubes? and so on, up to 20 cubes?
   b) How many cubes do you need to make exactly 1 prism? Exactly 2 prisms? Exactly 3 prisms? Exactly 4 prisms?
   c) What patterns do you see in your answers to part b?

8. a) Sketch 3 different rectangular prisms with volume 24 cm³.
   b) Which prism has the greatest surface area? The least surface area?
   c) Try to find a prism with a greater surface area. Describe the shape of this prism.
   d) Try to find a prism with a lesser surface area. Describe the shape of this prism.

Reflect

Suppose you know the volume of a rectangular prism.
How can you find its dimensions? Use words and pictures to explain.

Decoding Word Problems

1 READ the problem.

- Do I understand all the words?
- Highlight those words I don't understand. Whom can I ask?
- Are there key words that give me clues? Circle these words.

David, Jennifer, and Mark collect baseball cards. Jennifer has the most cards. She has 15 more cards than David. David has 4 times as many cards as Mark has today. Mark often loses some of his cards. This morning Mark has 18 cards. How many baseball cards does Jennifer have?

2 THINK about the problem.

- What is the problem about?
- What information am I given?
- Am I missing any information?
- Is this problem like another problem I have solved before?

STRATEGIES to consider . . .

Use a model.
Make an organized list.
Make a table.
Use logical reasoning.
Solve a simpler problem.
Draw a diagram.
Guess and check.
Use a pattern.
Work backward.
Draw a graph.

3 MAKE a plan.

- What strategy should I use?
- Would counters, geometric figures, other materials, calculators, and so on, help?

4 TRY out the plan.

- Is the plan working? Should I try something else?
- Have I shown *all* my work?

5 LOOK back.

- Does my answer make sense? How can I check?
- Is there another way to solve this problem?
- How do I know my answer is correct?
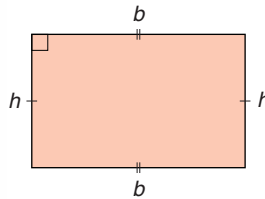- Have I answered the problem?

6 EXTEND.

- Ask "What if …?" questions. What if the numbers were different? What if there were more options?
- Make up similar problems of your own.
- STRETCH your thinking.
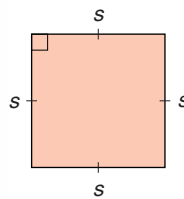
Unit Review

What Do I Need to Know?

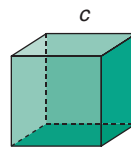☑ Perimeter of a rectangle:
$P = 2(b + h)$

☑ Area of a rectangle:
$A = bh$

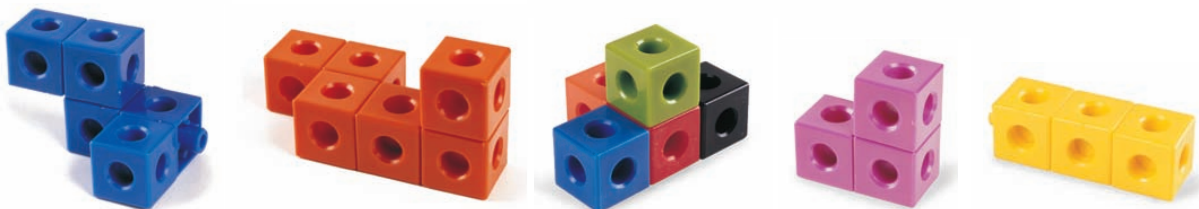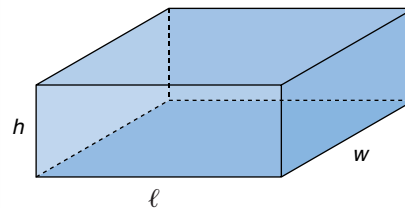☑ Perimeter of a square:
$P = 4s$

☑ Area of a square:
$A = s^2$

☑ Volume of a cube:
$V = c^3$

☑ Surface area of a cube:
$SA = 6c^2$

☑ Surface area of a rectangular prism:
$SA = 2lh + 2lw + 2hw$

☑ Volume of a rectangular prism:
$V = lwh$
or $V = Ah$, where $A = lw$

LESSON

3.1 **1.** Sketch the front, back, side, and top views of each object. Use square dot paper, plain paper, or *The Geometer's Sketchpad*.
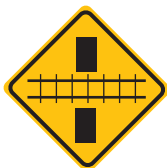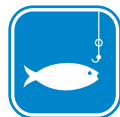
a)

b)

c)

3.2 **2. a)** Identify the view and the objects on each sign.

i) Railway Crossing Ahead

ii) Fishing Area

b) Design a new sign from a different view. Sketch your design.

3.3 **3.** This is a net for an octahedron.
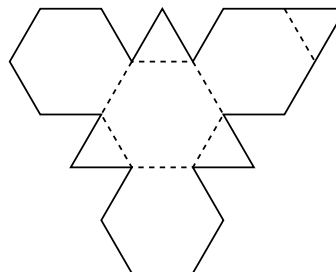
a) Fold a large copy of the net. Describe the object. Why do you think it is called an octahedron?

b) How is the object you built different from a regular octahedron?

3.4 **4.** Find the area of each figure.
a) a square with side length 7 cm
b) a rectangle with base 12 m and height 3 m

5. A cube has edge length c.
a) Write a formula for the surface area of the cube.
b) Use the formula to find the surface area when c is 4 cm.

6. A children's play area is 90 m long and 44 m wide. A fence will enclose the area.
a) How much fencing is needed?
b) Fencing comes in 12-m bundles. Each bundle costs $35. How much will the fence cost? Justify your answer.

3.5
3.6

7. Find the surface area and volume of each rectangular prism.

a)

6 m
2 m
3 m

b)

8 cm
3 cm
3 cm

c)

50 cm
50 cm
50 cm
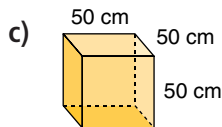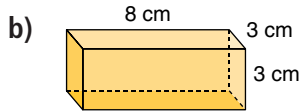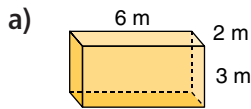
8. Elizabeth pastes wallpaper on 3 walls of her bedroom. She paints the 4th wall. This is one of the smaller walls. The dimensions of the room are 3 m by 5 m by 6 m. A roll of wallpaper covers about 5 m^2. A 4-L can of paint covers about 40 m^2.

a) How much wallpaper and paint should Elizabeth buy?

b) What assumptions do you make?

9. The surface area and volume of a cube have the same numerical value. Find the dimensions of this cube. How many answers can you find?

**10.** Sketch all possible rectangular prisms with a volume of 28 m^3. Each edge length is a whole number of metres. Label each prism with its dimensions. Calculate the surface area of each prism.

11. In *Gulliver's Travels* by Jonathan Swift, Gulliver visits a land where each of his dimensions is 12 times as large as that of the inhabitants.

a) Assume you can be modelled as a rectangular prism. Measure your height, width, and thickness.
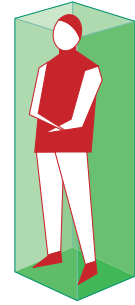
b) Suppose you were one of the inhabitants. What would Gulliver's dimensions be?

c) Use your dimensions. Calculate your surface area and volume.

d) Calculate Gulliver's surface area and volume.

12. The volume, *V*, and base area, *A*, of a rectangular prism are given. Find the height of each prism. Sketch the prism. What are possible dimensions for the prism? Explain.

a) $V = 18 \text{ m}^3, A = 6 \text{ m}^2$

b) $V = 60 \text{ cm}^3, A = 15 \text{ cm}^2$

13. Samya is making candles. She uses a 1-L milk carton as a mould. Its base is a 7-cm square. Samya pours 500 mL of wax into the carton. Recall that 1 mL = 1 cm^3. Approximately how tall will the candle be? Justify your answer.

Practice Test

1. Build the letter H with linking cubes.
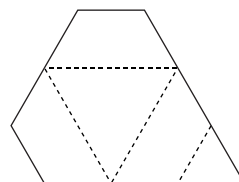   Draw the front, back, side, and top views.

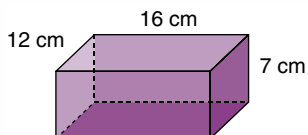2. Fold a copy of this net.
   a) Describe the object.
      What name could
      you give it?
      Justify your answer.
   b) Sketch the object.

3. Find the surface area and volume of this rectangular prism.

   12 cm   16 cm

   7 cm

4. Each edge length of a rectangular prism is 7 cm.
   a) Sketch the prism.
   b) Use formulas to find its volume and surface area.

5. Here are the dimensions of two designs for a sandbox for the
   primary playground. Each sandbox is a rectangular prism.
   One design is 3 m by 3 m by 30 cm.
   A second design is 3.25 m by 3.25 m by 25 cm.
   a) Sketch each sandbox. Include its dimensions.
   b) Compare the designs. For each design, calculate the area
      of material needed to build it and the volume of sand
      it will hold.
   c) Which sandbox should be built? Justify your answer.

6. Think about the nets you have folded and the solids
   you have made.
   Suppose you are a tent manufacturer.
   Which net would you use to make a tent?
   Explain why it would be a good tent design.

Suppose you are planning to sell baked goods at the
local charity bake sale.
You have to decide the selling price for your baked goods.
You want to make sure that the price covers the cost of the
ingredients and the packaging.

Decide on the baked goods you will sell.
For example, here is a recipe for Rice Krispies Treats.

Rice Krispies Treats

50 mL margarine
250 g marshmallows
1.5 L Rice Krispies

Melt the margarine.
Add marshmallows and
stir until melted.
Remove from heat.
Add Rice Krispies.
Stir until the Rice Krispies
are coated.
Press mixture into
greased 32-cm
by 23-cm pan.
When mixture is cool,
cut into squares.

Use this recipe or find your own recipe for treats.
How many batches of treats will you make?
Calculate how much of each ingredient you need.
Use grocery flyers to find how much the ingredients will cost.

1. Calculate the cost of 1 batch of treats.

2. How many treats will you sell in 1 package?
Design a box as the package for your treats.
Decide if the box is open and covered with plastic wrap,
or if the box is closed.
Draw a 3-D picture of your box.
Sketch a net for your box.

3. Calculate the surface area and volume of your box.
Suppose the cost of cardboard is 50¢/m^2.
How many boxes can you make from 1 m^2 of cardboard?
How much does each box cost?
What other costs are incurred to make the boxes?
What assumptions do you make?

4. Draw the top, front, back,
and side views of your box.

5. How much will you sell
1 box of treats for?
Justify your answer.
Show how the selling
price of the treats covers
the cost of all the
things you bought.

Reflect on the Unit

Write a paragraph to tell what you have learned about polyhedra in this unit.
Try to include something from each lesson in the unit.