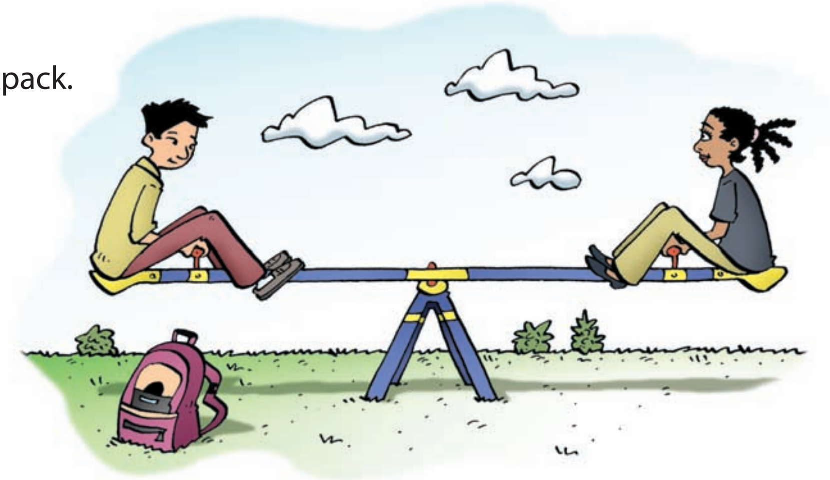
LESSON 7

Understanding Equality

Suppose the boy puts on his backpack.
What will happen?



Explore

You will need balance scales, counters, and drawings of balance scales.

➤ Choose 2 expressions from the box at the right. On a drawing of balance scales, write one expression in each pan.

➤ Suppose you were using real balance scales and counters for the numbers. Would the scales tilt to the left, to the right, or would they balance? How do you know? Use balance scales and counters to check.

➤ Repeat the steps above with different pairs of expressions. Find as many pairs of expressions as you can that balance.

Expressions

$4 + 5$	$8 + 3$
3×5	2×4
$17 - 10$	4×2
$18 \div 6$	$24 \div 4$
$15 - 8$	$30 \div 5$
$21 - 10$	$5 + 4$
$27 \div 9$	5×3

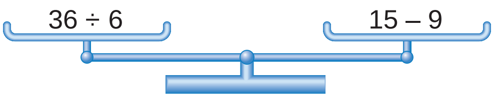
Show *and* Share

Share your work with another pair of classmates.
What strategies did you use to decide whether the scales balance or tilt?
What did you notice about the expressions $4 + 5$ and $5 + 4$, and 2×4 and 4×2?
What does it mean when the scales balance?

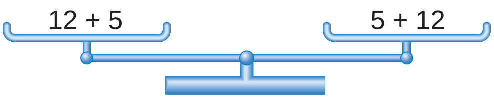
Lesson Focus | Understand equality and the commutative properties.

Connect

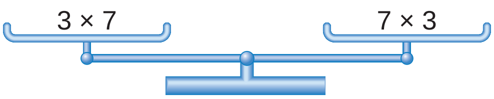
Each of the scales below are balanced.
For each balance scales, the expression in one pan is equal to the expression in the other pan.
We use the equals sign to show that the two expressions are equal.



$36 \div 6 = 6$ and
$15 - 9 = 6$
So, $36 \div 6 = 15 - 9$



$12 + 5 = 17$ and
$5 + 12 = 17$
So, $12 + 5 = 5 + 12$



$3 \times 7 = 21$ and
$7 \times 3 = 21$
So, $3 \times 7 = 7 \times 3$

➤ When we add 2 numbers, their order does not affect the sum.
The scales always balance.
This is called the **commutative property of addition**.
For example,
$3 + 2 = 2 + 3$
$114 + 35 = 35 + 114$

We can use variables to show this property for any pair of numbers we add:
$a + b = b + a$

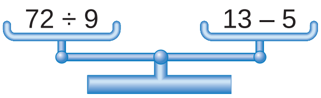
➤ Multiplication is also *commutative*.
When we multiply two numbers, their order does not affect the product.
For example,
$3 \times 2 = 2 \times 3$
$55 \times 8 = 8 \times 55$

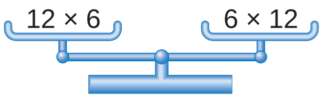
We can use variables to show this property for any pair of numbers we multiply:
$a \times b = b \times a$

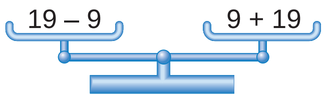
This illustrates the **commutative property of multiplication**.

Practice

1. Suppose you were using real balance scales.
Which scales below would balance?
How did you find out?

a) 

b) 

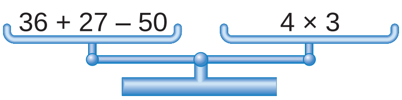
c) 

2. a) Write an expression with 2 numbers and one operation.
   b) Write 5 different expressions that equal your expression in part a.
   What strategy did you use to find the expressions?
   c) Suppose you used real balance scales.
   You put counters to represent 3 of the expressions in the left pan and 3 in the right pan. What would happen? How do you know?

3. Rewrite each expression using a commutative property.

a) $5 + 8$	b) 6×9	c) 11×7
d) $12 + 21$	e) $134 + 72$	f) 36×9

4. a) Are these scales balanced?



   b) If your answer is yes, why do you think so?
   If your answer is no, what could you do to balance the scales?
   Why would this work?

5. a) Addition and subtraction are inverse operations.
   Addition is commutative. Is subtraction commutative?
   Use an example to show your answer.
   b) Multiplication and division are inverse operations.
   Multiplication is commutative. Is division commutative?
   Use an example to show your answer.

Reflect

Are subtractions and division commutative operations?
Explain why or why not.

LESSON 8

Keeping Equations Balanced

Each of these tug-of-war teams has the same total mass.
Suppose a girl with mass 48 kg joins Team A.
What could be done to keep the match fair?

Explore

You will need counters.
Each group member chooses a different expression.

Expressions	
3×6	$17 - 5$
$3 + 5$	$24 \div 4$

➤ Write a different expression that is equal to the expression you chose.
Use the expressions to write an equation.

➤ Model the equation with counters.
How do the counters show the expressions are balanced?

➤ Find 4 different ways to adjust the original equation so that it remains balanced.
Use counters to model what you did each time.
Use symbols to record your work.

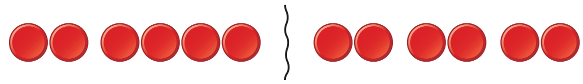
Show *and* Share

Share your work with another group of students.
What strategies did you use to keep the equation balanced?
Were you able to use each of the 4 operations?
If not, work together to try the operations that you did not use.

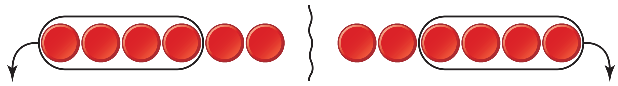
 LESSON FOCUS | Model and explain the meaning of the preservation of equality.

Connect

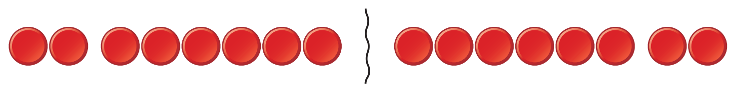
- Max started with this equation each time:
  $2 + 4 = 3 \times 2$
  He modelled it using counters.
  Each side has 6 counters.



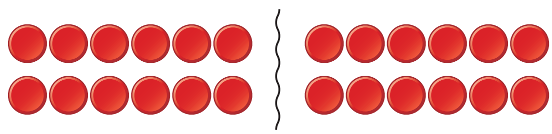
First, Max subtracted 4 from each side.
$6 - 4 = 6 - 4$
Each side now has 2 counters.



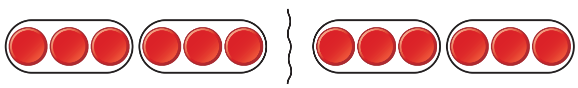
Second, Max added 2 to each side.
$6 + 2 = 6 + 2$
Each side now has 8 counters.



Third, Max multiplied each side by 2.
$6 \times 2 = 6 \times 2$
Each side now has 12 counters.



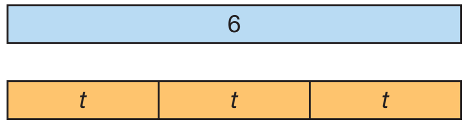
Fourth, Max divided each side into
2 equal groups.
$6 \div 2 = 6 \div 2$
Each group has 3 counters.



Whatever Max did to one side of the equation, he did to the other side, too.
Each time, the numbers of counters on both sides remained equal.
So, the equation remained balanced.

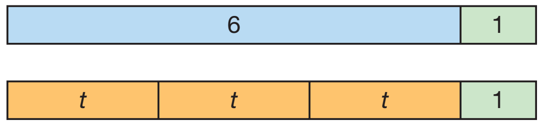
When each side of the equation is changed in the same way,
the values remain equal.
This is called the **preservation of equality**.
The same is true if one side of the equation is an expression containing a variable.

➤ Suppose we know $6 = 3t$.
We can model this equation with paper strips.

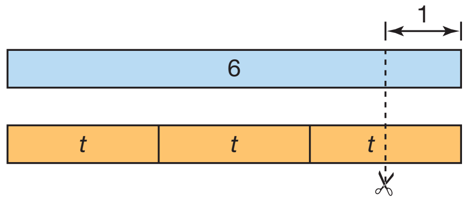


To preserve the equality, we can:

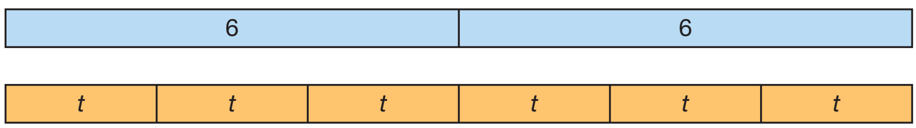
- Add the same number to each side.
  So, $6 + 1 = 3t + 1$



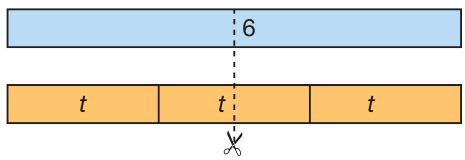
- Subtract the same number from each side.
  So, $6 - 1 = 3t - 1$



- Multiply each side by the same number.
  So, $2 \times 6 = 2 \times 3t$



- Divide each side by the same number.
  So, $6 \div 2 = 3t \div 2$



When we do the same to each side of an equation,
we produce an **equivalent form of the equation**.

So, $6 + 1 = 3t + 1$
$6 - 1 = 3t - 1$
$2 \times 6 = 2 \times 3t$
$6 \div 2 = 3t \div 2$
are all equivalent forms of the equation $6 = 3t$.

Practice

1. For each equation below:

- Model the equation with counters.
- Use counters to model the preservation of equality for addition.
- Draw a diagram to record your work.
- Use symbols to record your work.

a) $9 + 6 = 15$ **b)** $14 - 8 = 6$
c) $2 \times 5 = 10$ **d)** $15 \div 3 = 9 - 4$

2. For each equation below:
- Model the equation with counters.
- Use counters to model the preservation of equality for subtraction.
- Draw a diagram to record your work.
- Use symbols to record your work.

a) $7 + 8 = 15$ b) $12 - 7 = 5$
c) $3 \times 4 = 12$ d) $10 \div 5 = 9 - 7$

3. For each equation below:
- Model the equation with counters.
- Use counters to model the preservation of equality for multiplication.
- Draw a diagram to record your work.
- Use symbols to record your work.

a) $2 + 3 = 5$ b) $9 - 6 = 3$
c) $2 \times 4 = 8$ d) $12 \div 4 = 2 + 1$

4. For each equation below:
- Model the equation with counters.
- Use counters to model the preservation of equality for division.
- Draw a diagram to record your work.
- Use symbols to record your work.

a) $5 + 1 = 6$ b) $8 - 4 = 4$
c) $5 \times 2 = 10$ d) $16 \div 2 = 2 \times 4$

5. For each equation below:
- Apply the preservation of equality.
  Write an equivalent form of the equation.
- Use paper strips to check that equality has been preserved.

Try to use a different operation for each part.

a) $3b = 12$ b) $2t = 8$
c) $16 = 4s$ d) $15 = 5s$

How do you know that equality has been preserved each time?

Reflect

Talk to a partner. Tell your partner what you think the preservation of equality means. Describe how you could model the preservation of equality for each of the 4 operations.