UNIT 2 Integers

As of 2007, Mike Weir is the only Canadian to ever win the US Masters Golf Championship. Weir defeated Len Mattiace on the first extra hole of a playoff to win the 2003 Masters.

Here are seven players, in alphabetical order, and their leaderboard entries.

Player	Over/Under Par
Jim Furyk	–4
Retief Goosen	+1
Jeff Maggert	–2
Phil Mickelson	–5
Vijay Singh	–1
Mike Weir	–7
Tiger Woods	+2

- What was Weir's leaderboard entry?
- Order the entries from least to greatest.
- Why do you think golf is scored using integers?
- What other uses of integers do you know?

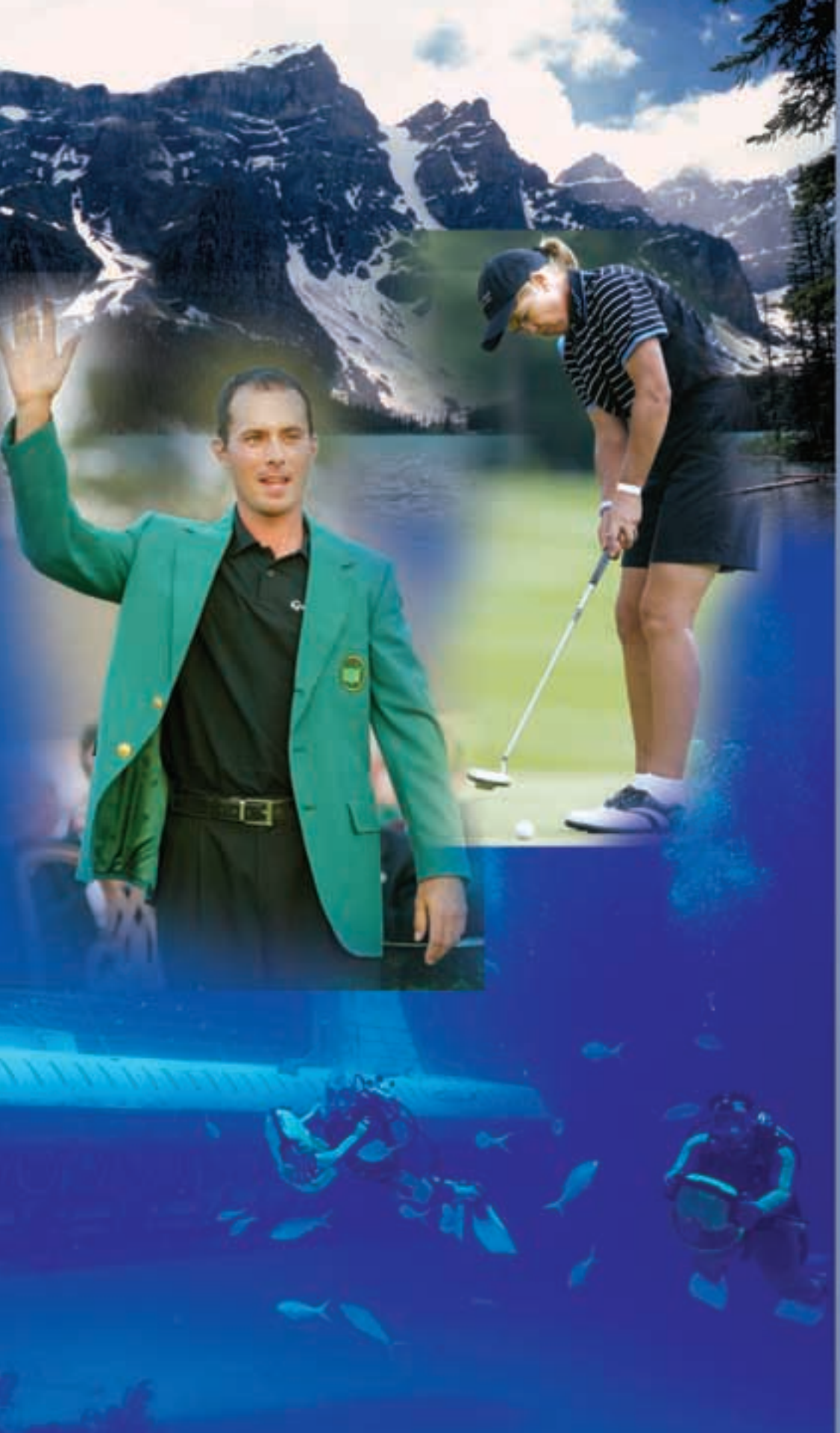
Par for the tournament is 288. Jim Furyk shot 284. His score in relation to par is 4 under, or –4.

What You'll Learn

- Multiply and divide integers.
- Use the order of operations with integers.
- Solve problems involving integers.

Why It's Important

We use integers in everyday life, when we deal with weather, finances, sports, geography, and science.



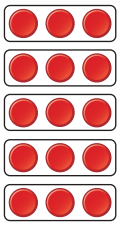
Key Words

- positive integer
- negative integer
- zero pair
- opposite integers
- zero property
- distributive property
- commutative property
- product
- quotient
- grouping symbol
- order of operations

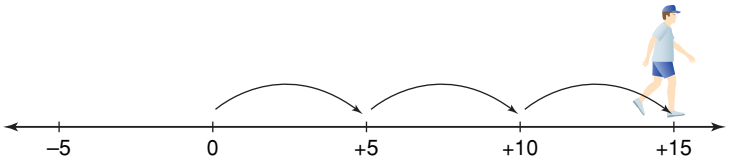
2.1 Using Models to Multiply Integers

Focus Use a model to multiply two integers.

We can think of multiplication as repeated addition.
5×3 is the same as adding five 3s: $3 + 3 + 3 + 3 + 3$
As a sum: $3 + 3 + 3 + 3 + 3 = 15$
As a product: $5 \times 3 = 15$



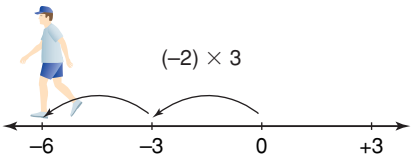
How can we think of 3×5?
One way is to use a number line.
Take 3 steps each of size 5.
So, $3 \times 5 = 15$

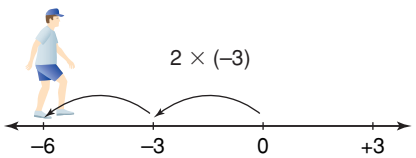


Investigate

Work with a partner.
You will need masking tape and a metre stick.
Make a large number line across the floor.
Divide the line into intervals of 15 cm.
Label the line from –15 to +15.

➤ Walk the line to multiply integers.
– Start at 0.
– For negative numbers of steps, face the negative end of the line before walking.
– For negative step sizes, walk backward.
For example:





➤ Choose 2 different positive integers less than +5.

➤ Find all possible products of the integers and their opposites.
Take turns. One partner walks the line to find each product.
The other partner records on a number line and writes the multiplication equation each time.

Reflect & Share

Share your results with the class. What patterns do you notice?
How can you predict the product of two integers?

Connect

Recall that we can use coloured tiles to model integers.
One yellow tile models +1, and one red tile models –1.

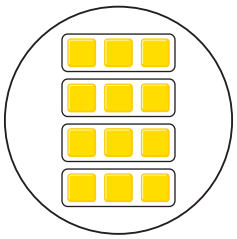

They combine to form a zero pair: (+1) + (–1) = 0

We can extend our use of coloured tiles to model the multiplication of two integers.
Let a circle represent the "bank." Start with the bank having zero value.
The first integer tells us to deposit (put in) or to withdraw (take out).
When the first integer is positive, put tiles in.
When the first integer is negative, take tiles out.
The second integer tells us what to put in or take out.

➤ Multiply: (+4) × (+3)
+4 is a positive integer.
+3 is modelled with 3 yellow tiles.
So, *put* 4 sets of 3 yellow tiles *into* the circle.

There are 12 yellow tiles in the circle.
They represent +12.
So, (+4) × (+3) = +12

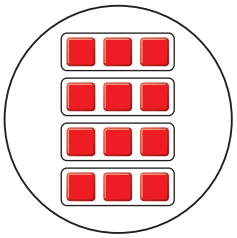
(+4) × (+3) = (+3) + (+3) + (+3) + (+3)
Make 4 deposits of +3.



➤ Multiply: (+4) × (–3)
+4 is a positive integer.
–3 is modelled with 3 red tiles.
So, *put* 4 sets of 3 red tiles *into* the circle.

There are 12 red tiles in the circle.
They represent –12.
So, (+4) × (–3) = –12

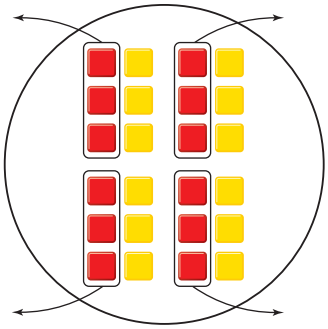
(+4) × (–3) = (–3) + (–3) + (–3) + (–3)
Make 4 deposits of –3.



➤ Multiply: (–4) × (–3)
–4 is a negative integer.
–3 is modelled with 3 red tiles.
So, *take* 4 sets of 3 red tiles *out* of the circle.
There are no red tiles to take out.

Make 4 withdrawals of –3.

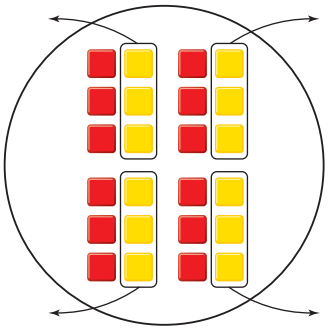
So, add zero pairs until there are enough red tiles to remove.
Add 12 zero pairs.
Take out 4 sets of 3 red tiles.



There are 12 yellow tiles left in the circle.
They represent +12.
So, $(-4) \times (-3) = +12$

➤ Multiply: $(-4) \times (+3)$
–4 is a negative integer.
+3 is modelled with 3 yellow tiles.
So, take 4 sets of 3 yellow tiles out of the circle.
There are no yellow tiles to take out.
So, add zero pairs until there are enough yellow tiles to remove.
Add 12 zero pairs.
Take out 4 sets of 3 yellow tiles.

Make 4 withdrawals of +3.

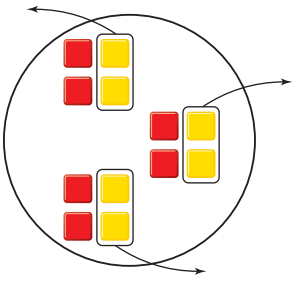


There are 12 red tiles left in the circle.
They represent –12.
So, $(-4) \times (+3) = -12$

Example 1

Use tiles to find: $(-3) \times (+2)$

A Solution



–3 is a negative integer.
+2 is modelled with 2 yellow tiles.
So, take 3 sets of 2 yellow tiles out of the circle.
Since there are no yellow tiles to take out, add 6 zero pairs.
Take out 3 sets of 2 yellow tiles.
6 red tiles remain.
They represent –6.
So, $(-3) \times (+2) = -6$

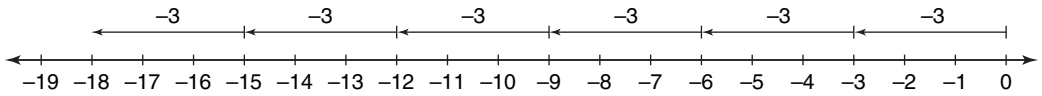
Example 2

The temperature fell 3°C each hour for 6 h.
Use an integer number line to find the total change in temperature.

A Solution

–3 represents a fall of 3°C.
+6 represents 6 h.
Using integers, we need to find: $(+6) \times (-3)$
$(+6) \times (-3) = (-3) + (-3) + (-3) + (-3) + (-3) + (-3)$
Use a number line. Start at 0.
Move 3 units left, 6 times.

To add a negative integer, move left on a number line.



So, $(+6) \times (-3) = -18$
The total change in temperature is –18°C.

Discuss the ideas

1. Which model do you find easiest to use to multiply integers?
2. What other models can you think of?
3. What do you notice about the effect of the order of the integers on the product?
4. Use a model to explain multiplication of an integer by 0.

Math Link

Sports

In hockey, each player has a plus/minus statistic. A player's plus/minus statistic increases by 1 when his team scores a goal while he is on the ice. A player's plus/minus statistic decreases by 1 when his team is scored against while he is on the ice. For example, a player begins a game with a plus/minus statistic of –7. During the game, his team scores 3 goals while he is on the ice and the opposing team scores 1 goal. What is the player's new plus/minus statistic?

Practice

Check

5. Write a multiplication expression for each repeated addition.

a) $(-1) + (-1) + (-1)$

b) $(-2) + (-2) + (-2) + (-2) + (-2)$

c) $(+11) + (+11) + (+11) + (+11)$

6. Write each multiplication expression as a repeated addition. Use coloured tiles to find each sum.

a) $(+7) \times (-4)$

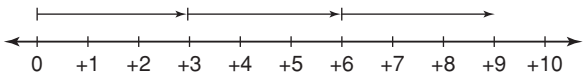
b) $(+6) \times (+3)$

c) $(+4) \times (+6)$

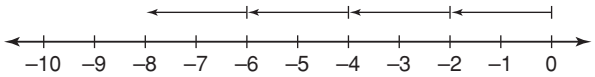
d) $(+5) \times (-6)$

7. Which integer multiplication does each number line represent? Find each product.

a)



b)



8. Use a number line. Find each product.

a) $(+6) \times (-1)$

b) $(+3) \times (+9)$

c) $(+2) \times (+6)$

d) $(+4) \times (-5)$

9. Which product does each model represent? Write a multiplication equation for each model.

a) Deposit 5 sets of 2 red tiles.

b) Deposit 5 sets of 2 yellow tiles.

c) Withdraw 7 sets of 3 red tiles.

d) Withdraw 9 sets of 4 yellow tiles.

e) Deposit 11 sets of 3 yellow tiles.

f) Withdraw 10 sets of 5 red tiles.

Apply

10. Use a circle and coloured tiles. Find each product. Sketch the tiles you used.

a) $(+1) \times (+5)$ **b)** $(+8) \times (+3)$

c) $(+7) \times (-2)$ **d)** $(+8) \times (-3)$

e) $(-5) \times (+6)$ **f)** $(-4) \times (-8)$

11. Use coloured tiles or a number line. Find each product.

a) $(+4) \times (+2)$ **b)** $(-4) \times (-2)$

c) $(+2) \times (+8)$ **d)** $(+5) \times (-6)$

e) $(-4) \times (+6)$ **f)** $(-7) \times (-3)$

12. The temperature rose 2°C each hour for 9 h. Use integers to find the total change in temperature.

13. Donovan was draining an above-ground swimming pool. The water level dropped 3 cm each hour for 11 h. Use integers to find the change in the water level after 11 h.

14. Lissa used the expression $(+8) \times (-6)$ to solve a word problem.

a) What might the word problem have been? Solve the problem.

b) Compare your word problem with those of your classmates. Which is your favourite problem?

15. **Assessment Focus** How many different ways can you model the product $(-7) \times (-8)$?
Show each strategy. Which strategy do you prefer? Explain your choice.

16. Rema was playing a board game. She moved back 4 spaces on each of 4 consecutive turns. Use integers to find her total change in position.

17. Ellen's dad spends $5 a week on newspapers.

a) How much less newspaper money will he have 8 weeks from now?

b) How much more newspaper money did he have 2 weeks ago?

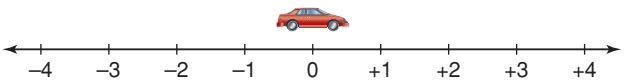
Draw a model to represent each answer. Write the equation that each model represents.

18. A toy car travels along a number line marked in centimetres. A distance of 1 cm to the right is represented by +1. A distance of 1 cm to the left is represented by –1. The car moves 4 cm to the left each second.

a) The car is at 0 now. Where will the car be 10 s from now?

b) Where was the car 3 s ago?

c) How can you use an operation with integers to answer parts a and b?



19. Hugh used the expression $(-7) \times (+6)$ to solve a word problem. What might the word problem have been? Solve the problem.

20. **Take It Further** Use coloured tiles or a number line. Find each product.

a) $(+3) \times (-2) \times (+4)$

b) $(-5) \times (-1) \times (+3)$

c) $(-5) \times (-2) \times (-3)$

d) $(+2) \times (-3) \times (-6)$

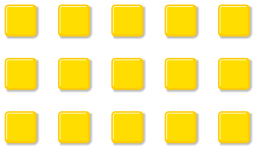
Reflect

How did your knowledge of adding integers help you with this lesson?
How did your knowledge of opposites change with this lesson?

2.2 Developing Rules to Multiply Integers

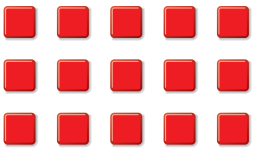
Focus Use patterns to develop rules for multiplying integers.

We can write the number of tiles in this array in two ways.
As a sum:
$(+5) + (+5) + (+5) = +15$



As a product:
$(+3) \times (+5) = +15$

How can you use integers to write the number of tiles in this array in two ways?



Investigate

Work with a partner.

Your teacher will give you a large copy of this multiplication table.

Fill in the products that you know best.
Use any patterns you see to help you complete the table.

Second Number

First Number

×	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5
−5											
−4											
−3											
−2											
−1											
0											
+1											
+2											
+3											
+4											
+5											

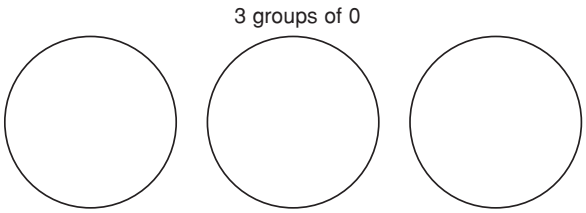
Reflect & Share

Compare your completed table with that of another pair of classmates.
Explain a strategy you could use to multiply any negative integer by any positive integer.
Explain a strategy you could use to multiply any two negative integers.

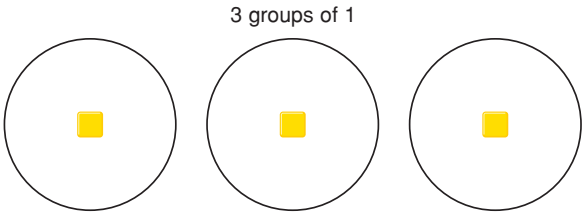
Connect

These properties of whole numbers are also properties of integers.

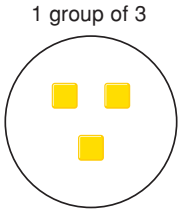
Multiplying by 0 (Zero property)
$3 \times 0 = 0$ and $0 \times 3 = 0$
So, $(-3) \times 0 = 0$ and $0 \times (-3) = 0$



Multiplying by 1 (Multiplicative Identity)
$3 \times 1 = 3$ and $1 \times 3 = 3$
So, $(-3) \times (+1) = -3$ and $(+1) \times (-3) = -3$



Since multiplying by 1 does not change the identity of a number, we call 1 the *multiplicative identity*.

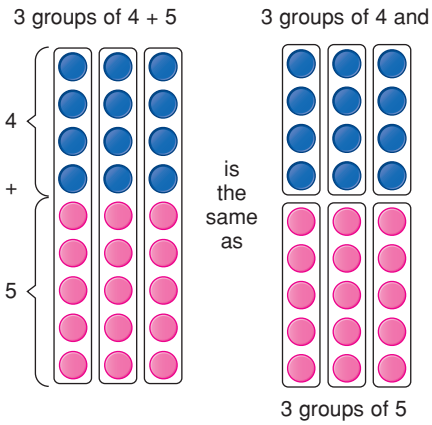


Commutative Property
$3 \times 4 = 12$ and $4 \times 3 = 12$
So, $(-3) \times (+4) = -12$ and $(+4) \times (-3) = -12$

Distributive Property

$$\begin{aligned} 3 \times (4 + 5) &= 3 \times 4 + 3 \times 5 \\ &= 12 + 15 \\ &= 27 \end{aligned}$$

$$\begin{aligned} \text{So, } (+3) \times [(-4) + (-5)] &= [(+3) \times (-4)] + [(+3) \times (-5)] \\ &= (-12) + (-15) \\ &= -27 \end{aligned}$$



Example 1

Find each product.

a) $(-9) \times (+4)$ **b)** $(-4) \times (-9)$ **c)** $(+4) \times (+9)$

A Solution

a) Multiply the numbers as if they were positive.
$9 \times 4 = 36$
The integers have opposite signs, so the product is negative.
So, $(-9) \times (+4) = -36$

b) The integers have the same sign, so the product is positive.
So, $(-4) \times (-9) = +36$

c) The integers have the same sign, so the product is positive.
So, $(+4) \times (+9) = +36$

Example 2

Find the product: $(+20) \times (-36)$

A Solution

$$\begin{aligned}(+20) \times (-36) &= (+20) \times [(-30) + (-6)] \\ &= [(+20) \times (-30)] + [(+20) \times (-6)] \\ &= (-600) + (-120) \\ &= -720\end{aligned}$$

So, $(+20) \times (-36) = -720$

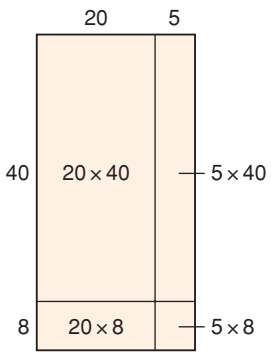
Write –36 in expanded form.
Use the distributive property.

Example 3

Find the product: $(-25) \times (-48)$

A Solution

Multiply the numbers as if they were positive: 25×48
Use a rectangle model.



$$\begin{aligned}(25) \times (48) &= (20 \times 40) + (5 \times 40) + (20 \times 8) + (5 \times 8) \\ &= 800 + 200 + 160 + 40 \\ &= 1200\end{aligned}$$

The integers have the same sign, so the product is positive.
So, $(-25) \times (-48) = +1200$

When we write the product of integers, we do not need to write the multiplication sign. That is, we may write $(-8) \times (-9)$ as $(-8)(-9)$.

Discuss the ideas

1. Think about the work of this lesson and the previous lesson. What is the sign of the product when you multiply 2 integers:
   - if both integers are positive?
   - if one integer is positive and the other integer is negative?
   - if both integers are negative?
2. Explain your strategy to multiply 2 integers.

Practice

Check

3. Will each product be positive or negative? How do you know?
   a) $(-6) \times (+2)$ b) $(+6) \times (+4)$
   c) $(+4) \times (-2)$ d) $(-7) \times (-3)$

4. Find each product.
   a) $(+8)(-3)$ b) $(-5)(-4)$
   c) $(-3)(+9)$ d) $(+7)(-6)$
   e) $(+10)(-3)$ f) $(-7)(-6)$
   g) $(0)(-8)$ h) $(+10)(-1)$
   i) $(-7)(-8)$ j) $(+9)(-9)$

5. a) Find the product of each pair of integers.
   i) $(+3)(-7)$ and $(-7)(+3)$
   ii) $(+4)(+8)$ and $(+8)(+4)$
   iii) $(-5)(-9)$ and $(-9)(-5)$
   iv) $(-6)(+10)$ and $(+10)(-6)$
   b) Use the results of part a. Does the order in which integers are multiplied affect the product? Explain.

6. Find each product.
   a) $(+20) \times (+15)$ b) $(-30) \times (-26)$
   c) $(+50) \times (-32)$ d) $(-40) \times (+21)$
   e) $(-60) \times (+13)$ f) $(+80) \times (-33)$
   g) $(+70) \times (+47)$ h) $(-90) \times (-52)$

Apply

7. Find each product.
   a) $(+25) \times (-12)$ b) $(-45) \times (+21)$
   c) $(-34) \times (-16)$ d) $(-37) \times (+18)$
   e) $(+17)(+13)$ f) $(+84)(-36)$
   g) $(-51)(-25)$ h) $(+29)(+23)$

8. Copy each equation. Replace $\square$ with an integer to make the equation true.
   a) $(+5) \times \square = +20$
   b) $\square \times (-9) = +27$
   c) $(-9) \times \square = -54$
   d) $\square \times (-3) = +18$
   e) $\square \times (+5) = -20$
   f) $\square \times (-12) = +144$
   g) $\square \times (-6) = +180$
   h) $\square \times (-4) = +24$

9. Write the next 3 terms in each pattern. Then write the pattern rule.

a) +1, +2, +4, +8, …

b) +1, –6, +36, –216, …

c) –1, +3, –9, +27, …

d) –4, +4, –4, +4, …

10. Gaston withdrew $26 from his bank account each week for 17 weeks. Use integers to find the total amount Gaston withdrew over the 17 weeks. Show your work.

11. **Assessment Focus** Use the integers: –5, +9, –8, +4, –2

a) Which two integers have the greatest product?

b) Which two integers have the least product?

c) Provide a convincing argument that your answers to parts a and b are correct.

12. **a)** Find each product. Then use a calculator to extend the pattern 4 more rows.

i) (–2)(–3)

ii) (–2)(–3)(–4)

iii) (–2)(–3)(–4)(–5)

iv) (–2)(–3)(–4)(–5)(–6)

b) Use the results in part a.

i) What is the sign of a product when it has an even number of negative factors? Explain.

ii) What is the sign of a product when it has an odd number of negative factors? Explain.

c) Investigate what happens when a product has positive and negative factors. Do the rules in part b still apply? Explain.

13. Amelie was doing a math question. The answer she got did not match the answer in the answer key. So, she asked a friend to look at her work.

$$
\begin{aligned}
&(+60) \times (-18) \\
&\quad = (+60) \times [(-20) + (+2)] \\
&\quad = [(+60) \times (-20)] + [(+60) \times (+2)] \\
&\quad = (+1200) + (+120) \\
&\quad = +1320
\end{aligned}
$$

a) What was Amelie's error?

b) Correct Amelie's error. What is the correct answer?

14. Gavin used the expression $(+15) \times (-8)$ to solve a word problem. What might the word problem have been? Solve the problem.

15. Explain why an integer multiplied by itself can never result in a negative product.

16. Bridget used the expression $(-12) \times (+7)$ to solve a word problem. What might the word problem have been? Solve the problem.

17. Write –36 as the product of two or more integer factors. Do this as many different ways as you can. Show your work.

18. The product of two integers is –144. The sum of the integers is –7. What are the two integers?

19. **Take It Further** When you multiply two natural numbers, the product is never less than either of the two numbers. Is the same statement true for the product of any two integers? Investigate, then write what you find out.

Natural numbers are 1, 2, 3, 4, …, and so on.

20. **Take It Further** The product of two integers is between +160 and +200. One integer is between –20 and –40.

a) What is the greatest possible value of the other integer?

b) What is the least possible value of the other integer?

21. **Take It Further** How can you use the rules for multiplying two integers to multiply more than two integers? Use an example to illustrate your strategy.

Reflect

Suppose your friend missed this lesson. Explain to her how to multiply two integers. Use examples in your explanation.

What's My Product?

Game

In this game, a black card represents a positive integer; for example, the 5 of spades is +5.
A red card represents a negative integer; for example, the 6 of hearts is –6.

HOW TO PLAY

1. Remove the face cards (Jacks, Queens, Kings) from the deck. Use the remaining cards. An ace is 1.

2. Shuffle the cards. Place them face down in a pile. Players take turns to turn over the top 2 cards, then find the product. The person with the greater product keeps all 4 cards.

3. If there is a tie, each player turns over 2 more cards and the player with the greater product keeps all 8 cards.

4. Play continues until all cards have been used. The winner is the player with more cards.

YOU WILL NEED

A standard deck of playing cards

NUMBER OF PLAYERS

2

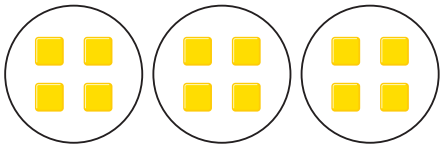
GOAL OF THE GAME

To have more cards

2.3 Using Models to Divide Integers

Focus Use a model to divide integers.

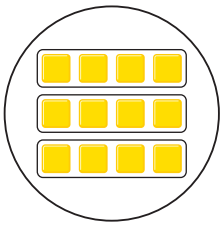
We can think of division as the opposite of multiplication.
$12 \div 4 = ?$
This can mean how many sets of 4 will give a product of 12:
$? \times 4 = 12$



There are 3 sets of 4.
So, $3 \times 4 = 12$

We can use a "bank" model to multiply 2 integers.

- A circle represents the "bank."
  We start with the bank having zero value.
- The first integer tells us to deposit (put in) or to withdraw (take out).
- The second integer tells us what to put in or take out.
- We can use this model to multiply 3×4.



Make 3 deposits of 4 yellow tiles.
There are 12 yellow tiles.
So, $3 \times 4 = 12$

How can we use this model to find $12 \div 4$?

Investigate

Work with a partner.
You will need coloured tiles.
Choose 2 positive integers between 1 and 20 whose quotient is an integer.
For example: 12 and 3, since $12 \div 3 = 4$
Use these integers and their opposites.
Write all possible division expressions.

A quotient is the number that results from the division of one number by another.

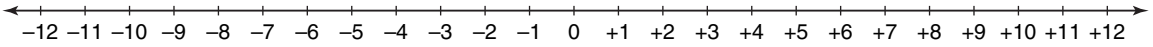
Use the tiles and a "bank" model to divide.
Sketch the tiles you used in each case.
Write a division equation each time.

Reflect & Share

Share your results with the class.
What patterns do you notice?
How can you predict the quotient of two integers?

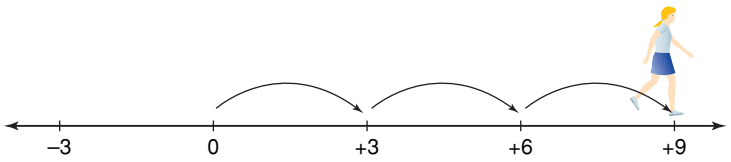
Connect

We can extend the use of a number line to model the division of two integers.



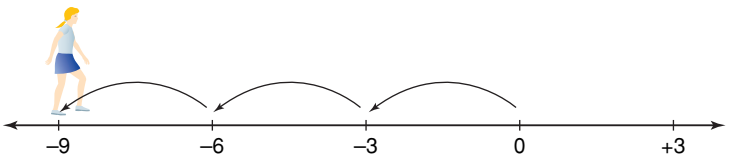
Visualize walking the line to divide integers.
This time, the direction you end up facing determines the sign of the quotient.

➤ Divide: (+9) ÷ (+3)
We need to find how many steps of +3 make +9.
The step size, +3, is positive; so, walk forward.
Start at 0. Take steps forward to end up at +9.



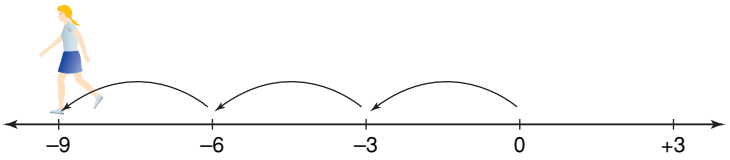
We took 3 steps. We are facing the positive end of the line.
So, (+9) ÷ (+3) = +3

➤ Divide: (–9) ÷ (–3)
We need to find how many steps of –3 make –9.
The step size, –3, is negative; so, walk backward.
Start at 0. Take steps backward to end up at –9.



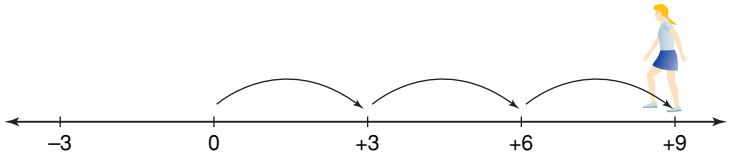
We took 3 steps. We are facing the positive end of the line.
So, (–9) ÷ (–3) = +3

➤ Divide: (–9) ÷ (+3)
We need to find how many steps of +3 take us to –9.
The step size, +3, is positive; so, walk forward.
Start at 0. To end up at –9, we took 3 steps forward.



We are facing the negative end of the line.
So, (–9) ÷ (+3) = –3

➤ Divide: (+9) ÷ (–3)
We need to find how many steps of –3 take us to +9.
The step size, –3, is negative; so, walk backward.
Start at 0. To end up at +9, we took 3 steps backward.



We are facing the negative end of the line.
So, (+9) ÷ (–3) = –3

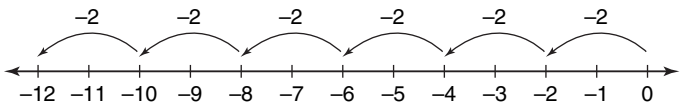
Example 1

The 1850-km Iditarod dogsled race lasts from 10 to 17 days.
One night, the temperature fell 2°C each hour for a total change of –12°C.
Use integers to find how many hours this change in temperature took.

A Solution

–2 represents a fall of 2°C.
–12 represents a change of –12°C.
Using integers, we need to find how many –2s take us to –12; that is, (–12) ÷ (–2).

Start at 0.
Move 2 units left.
Continue to move 2 units left until you reach –12.



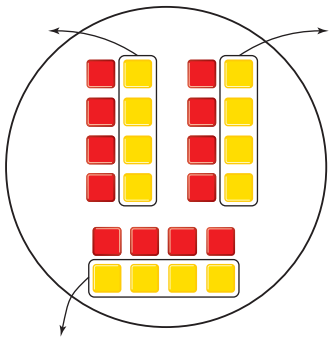
Six moves of 2 units left were made.
So, (–12) ÷ (–2) = +6

Example 2

Use a model to find the quotient: (–12) ÷ (+4)

A Solution

Divide: (–12) ÷ (+4)
How many groups of +4 will make –12?
Use coloured tiles and the "bank" model.



Start with a value of 0 in the circle.
To get a product of –12, 12 red tiles must be left in the circle.
So, model 0 with 12 zero pairs.
+4 is modelled with 4 yellow tiles.
Take out sets of 4 yellow tiles.

3 sets were removed.
So, (–12) ÷ (+4) = –3

The negative sign in the quotient indicates removal.

Discuss the ideas

1. Which model do you prefer to use to divide integers?
2. How can you use the inverse operation to check your answers?

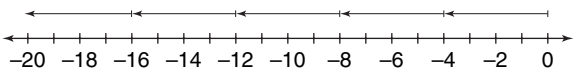
Practice

Check

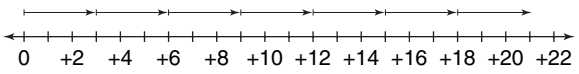
3. Write a related multiplication equation for each division equation.
   a) (+25) ÷ (+5) = +5
   b) (+24) ÷ (–2) = –12
   c) (–14) ÷ (–7) = +2
   d) (–18) ÷ (+6) = –3

4. Which integer division does each number line represent? Find each quotient.

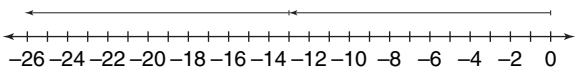
a)



b)



c)



5. Enrico walked a number line to model a division. He started at 0. Enrico took steps forward of size 4. He ended up at –24. Which division did Enrico model? How did you find out?

6. Use a number line. Find each quotient.

a) (+8) ÷ (+1) **b)** (–6) ÷ (–2)
c) (–16) ÷ (+8) **d)** (–3) ÷ (–1)
e) (+15) ÷ (–3) **f)** (–20) ÷ (+2)

7. **a)** How many sets?

i) 12 yellow tiles grouped in sets of 6
ii) 15 red tiles grouped in sets of 3

b) How many in each set?

i) 8 yellow tiles shared among 2 sets
iii) 21 red tiles shared among 7 sets

Apply

8. Use coloured tiles to represent each division. Find each quotient. Sketch the tiles you used.

a) (+18) ÷ (+6) **b)** (–18) ÷ (+9)
c) (–16) ÷ (–4) **d)** (+21) ÷ (–7)
e) (+15) ÷ (–5) **f)** (–16) ÷ (–8)

9. Use coloured tiles, a number line, or another model to show your thinking clearly. Find each quotient.

a) (+8) ÷ (+4) **b)** (–8) ÷ (–4)
c) (+8) ÷ (–4) **d)** (–8) ÷ (+4)

Compare the quotients. What do you notice?

10. Use coloured tiles, a number line, or another model to show your thinking clearly. Find each quotient.

a) (+24) ÷ (+8) **b)** (–20) ÷ (–5)
c) (+28) ÷ (–7) **d)** (–25) ÷ (+5)
e) (–14) ÷ (+2) **f)** (+18) ÷ (–9)

11. The temperature rose 3°C each hour for a total change of +12°C. Use integers to find the number of hours the change in temperature took.

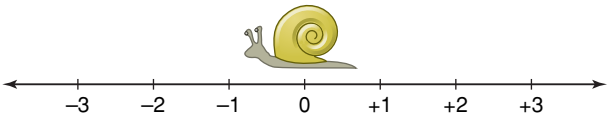
12. The temperature fell 4°C each hour for a total change of –20°C. Use integers to find the number of hours the change in temperature took.

13. A submarine was at the surface of the ocean. It made 4 identical plunges in a row. Its final depth was 148 m below sea level. What was the depth of each plunge?

14. **Assessment Focus** Heather used the expression (+45) ÷ (–5) to solve a word problem. What might the word problem have been? Show as many different ways as you can to solve the problem.

15. Maddie used the expression (–12) ÷ (+6) to solve a word problem. What might the word problem have been? Solve the problem.

16. A snail travels along a number line marked in centimetres. A distance of 1 cm to the right is represented by +1. A distance of 1 cm to the left is represented by –1. The snail moves 6 cm to the left each minute.



a) The snail is at 0 now. After how many minutes will the snail be at –36 on the number line?

b) When was the snail at +18 on the number line?

Draw a model to represent each answer. Write a division equation for each model.

17. **Take It Further** Abraham used the Internet to find the low temperature in six Western Canadian cities on a particular day in January. He recorded the temperatures in a table.

City	Low Temperature (°C)
Whitehorse	–10
Iqaluit	–7
Vancouver	+9
Edmonton	–1
Winnipeg	–9
Saskatoon	–6

a) Find the mean low temperature for these cities on that day.

b) The low temperature in Regina for the same day was added to the table. The mean low temperature for the seven cities was –3°C. What was the temperature in Regina?

Communicate your thinking clearly.

18. **Take It Further** Reena deposits $4 into her savings account each week. Today, Reena's account has a balance of $16.

a) How many weeks from now will Reena's account have a balance of $40?

b) What was the balance in Reena's account 2 weeks ago?

Explain how you can use integers to model each situation.

Reflect

How is the division of integers similar to the division of whole numbers? How is it different? Use examples to explain.

Mid-Unit Review

LESSON

2.1

1. Use a model to represent each product. Draw the model you used each time.

a) $(-9) \times (+4)$ **b)** $(-7) \times (-5)$

c) $(+4) \times (+8)$ **d)** $(+3) \times (-5)$

2. A glacier retreated about 2 m per day for 7 days. Use integers to find the total change in the length of the glacier.

3. The temperature rose 4°C each hour for 5 h. Use integers to find the total change in temperature.

2.2

4. Will each product be positive or negative? How do you know?

a) $(-8) \times (+5)$ **b)** $(-5) \times (-3)$

c) $(+12) \times (-4)$ **d)** $(+8) \times (+9)$

5. Find each product in question 4.

6. Find each product.

a) $(-20)(+14)$

b) $(-19)(-24)$

c) $(+40)(+27)$

d) $(+13)(-31)$

7. A swimming pool drains 35 L of water in 1 min. Find how much water drained out of the pool in 30 min. How can you model this situation with integers?

8. Copy each equation. Replace $\square$ with an integer to make the equation true.

a) $(+4) \times \square = -32$

b) $\square \times (-6) = +54$

c) $(-8) \times \square = -56$

d) $\square \times (-1) = +12$

2.3

9. Write 2 related multiplication equations for each division equation.

a) $(+27) \div (+3) = +9$

b) $(+14) \div (-7) = -2$

c) $(-21) \div (-3) = +7$

d) $(-26) \div (+2) = -13$

10. Use coloured tiles, a number line, or another model. Find each quotient.

a) $(+20) \div (+4)$ **b)** $(-24) \div (-6)$

c) $(+32) \div (-8)$ **d)** $(-36) \div (+4)$

11. The water level in a well dropped 5 cm each hour. The total drop in the water level was 30 cm. Use integers to find how long it took for the water level to change.

12. Maurice used the expression $(-18) \div (+3)$ to solve a word problem. What might the word problem have been? Solve the problem.

13. Explain how you can use a number line to model the quotient of $(+64) \div (-8)$.

2.4 Developing Rules to Divide Integers

Focus Use patterns to develop the rules for dividing integers.

For any multiplication fact with two different whole number factors, you can write two related division facts.
For example: $9 \times 7 = 63$
What are the related division facts? What strategies did you use?

We can apply the same strategies to multiplication facts with two different integer factors.

Investigate

Work in groups of 4.
Choose 2 positive integers and 2 negative integers.
Express each integer as a product of integers.
Do this in two ways.
For each product, write two related division facts.

Reflect & Share

Compare your division facts with those of another group of classmates. Generalize a rule you can use to determine the sign of the quotient when you divide:

- a positive integer by a positive integer
- a positive integer by a negative integer
- a negative integer by a positive integer
- a negative integer by a negative integer

For each rule, create two examples of your choice.

Connect

To divide integers, we can use the fact that division is the inverse of multiplication.

➤ We know that:
$(+5) \times (+3) = +15$

So, $(+15) \div (+5) = +3$ and $(+15) \div (+3) = +5$

dividend **divisor** **quotient**

When the dividend and the divisor are positive,
the quotient is positive.

➤ We know that:
$(-5) \times (+3) = -15$

So, $(-15) \div (+3) = -5$ and $(-15) \div (-5) = +3$

The dividend is negative and
the divisor is positive.
The quotient is negative.

Both the dividend and
the divisor are negative.
The quotient is positive.

➤ We know that:
$(-5) \times (-3) = +15$

So, $(+15) \div (-5) = -3$ and $(+15) \div (-3) = -5$

When the dividend is positive and the divisor is negative,
the quotient is negative.

A division expression can be written with a division sign: $(-48) \div (-6)$;
or, as a fraction: $\frac{-48}{-6}$

When the expression is written as a fraction, we do not need to use brackets.
The fraction bar acts as a **grouping symbol**.
A grouping symbol keeps terms together, just like brackets.

Example 1

Divide.

a) $(-8) \div (-4)$ **b)** $\frac{-9}{+3}$

A Solution

a) Divide the numbers as if they were positive.

$8 \div 4 = 2$

The integers have the same sign; so, the quotient is positive.

$(-8) \div (-4) = +2$

b) The integers have opposite signs; so, the quotient is negative.

$\frac{-9}{+3} = -3$

Example 2

Divide: $\frac{+96}{-6}$

A Solution

Divide as you would whole numbers.

$6\overline{)9^{3}6}$ $1\,6$

The integers have opposite signs; so, the quotient is negative.

So, $\frac{+96}{-6} = -16$

Example 3

Shannon made withdrawals of $14 from her bank account.
She withdrew a total of $98.
Use integers to find how many withdrawals Shannon made.

A Solution

–14 represents a withdrawal of $14.
–98 represents a total withdrawal of $98.
Divide to find the number of withdrawals.
Since each integer has 2 digits, use a calculator.
$(-98) \div (-14) = +7$
Shannon made 7 withdrawals of $14.

Discuss the ideas

1. Think about the work of this lesson and the previous lesson. What is the sign of the quotient when you divide 2 integers:
   - if both integers are positive?
   - if one integer is positive and the other integer is negative?
   - if both integers are negative?
2. Explain your strategy to divide 2 integers.
3. How do these rules compare to the rules for multiplying integers?

Practice

Check

4. Will each quotient be positive or negative? How do you know?

a) $(-45) \div (+5)$

b) $(+16) \div (+8)$

c) $(+24) \div (-2)$

d) $(-30) \div (-6)$

5. Find each quotient.

a) $(+12) \div (+4)$ **b)** $(-15) \div (-3)$

c) $(-18) \div (+9)$ **d)** $(+81) \div (-9)$

e) $(+72) \div (-8)$ **f)** $(-64) \div (-8)$

g) $(-14) \div (+1)$ **h)** $(+54) \div (-6)$

i) $(-27) \div (-3)$ **j)** $(+32) \div (+4)$

6. Copy and continue each pattern until you have 8 rows. What does each pattern illustrate?

a) $(-12) \div (+3) = -4$
$(-9) \div (+3) = -3$
$(-6) \div (+3) = -2$
$(-3) \div (+3) = -1$

b) $(+25) \div (-5) = -5$
$(+15) \div (-3) = -5$
$(+5) \div (-1) = -5$
$(-5) \div (+1) = -5$

c) $(+8) \div (+2) = +4$
$(+6) \div (+2) = +3$
$(+4) \div (+2) = +2$
$(+2) \div (+2) = +1$

d) $(+14) \div (+7) = +2$
$(+10) \div (+5) = +2$
$(+6) \div (+3) = +2$
$(+2) \div (+1) = +2$

e) $(-14) \div (+7) = -2$
$(-10) \div (+5) = -2$
$(-6) \div (+3) = -2$
$(-2) \div (+1) = -2$

f) $(-10) \div (-5) = +2$
$(-5) \div (-5) = +1$
$(0) \div (-5) = 0$
$(+5) \div (-5) = -1$

7. **a)** Use each multiplication fact to find a related quotient.

i) Given $(+8) \times (+3) = +24$, find $(+24) \div (+3) = \square$.

ii) Given $(-5) \times (-9) = +45$, find $(+45) \div (-9) = \square$.

iii) Given $(-7) \times (+4) = -28$, find $(-28) \div (+4) = \square$.

b) For each division fact in part a, write a related division fact.

Apply

8. Write 2 related division facts for each multiplication fact.

a) $(-6) \times (+5) = -30$

b) $(+7) \times (+6) = +42$

c) $(+9) \times (-4) = -36$

d) $(-4) \times (-8) = +32$

9. Divide.

a) $\frac{-20}{-5}$ **b)** $\frac{+21}{-7}$

c) $\frac{-36}{+4}$ **d)** $\frac{0}{-8}$

10. Copy each equation. Replace $\square$ with an integer to make the equation true.

a) $(+25) \div \square = +5$

b) $\square \div (-9) = +10$

c) $(-63) \div \square = -7$

d) $\square \div (-3) = +7$

e) $\square \div (+5) = -12$

f) $\square \div (-7) = -7$

g) $\square \div (-6) = +8$

h) $\square \div (-4) = -11$

11. Nirmala borrowed \$7 every day. She now owes \$56. For how many days did Nirmala borrow money?

a) Write this problem as a division expression using integers.

b) Solve the problem.

12. The temperature dropped a total of 15°C over a 5-h period. The temperature dropped by the same amount each hour. Find the hourly drop in temperature.

a) Write this problem as a division expression using integers.

b) Solve the problem.

13. Winnie used the money in her savings account to pay back a loan from her mother. Winnie paid back her mother in 12 equal weekly payments. Over the 12 weeks, the balance in Winnie's savings account decreased by \$132.
By how much did her balance change each week?

14. An equestrian was penalized a total of 24 points over a number of performances. The mean number of points lost per performance was –6.
How many performances did the equestrian make?

a) Write this problem as a division expression using integers.

b) Solve the problem.

15. Write the next three terms in each pattern. What is each pattern rule?

a) +1, –3, +9, –27, …

b) +6, –12, +18, –24, …

c) +5, +20, –10, –40, +20, +80, …

d) +128, –64, +32, –16, …

e) –1 000 000, +100 000, –10 000, +1000, …

16. Assessment Focus Suppose you divide two integers. The quotient is an integer. When is the quotient:

a) less than both integers?
b) greater than both integers?
c) between the two integers?
d) equal to +1?
e) equal to –1?
f) equal to 0?

Use examples to illustrate your answers. Show your work.

17. Find all the divisors of –32. Write a division equation each time.

18. Divide.

a) (+60) ÷ (–12) **b)** (–90) ÷ (–15)
c) (–77) ÷ (+11) **d)** (–80) ÷ (–20)
e) (+56) ÷ (+14) **f)** (+90) ÷ (–18)

19. Calculate the mean of these bank deposits and withdrawals. Show your work.
+$36, –$20, –$18, +$45, +$27, –$16

20. Bea used the expression (+78) ÷ (–13) to solve a word problem. What might the word problem have been? Solve the problem.

21. Take It Further A warm front caused the temperature to rise 20°C over a 10-h period.

a) What was the average rise in temperature per hour?
b) After the rise in temperature, the precipitation that falls is snow. What could the temperature have been 10 h ago?

How many different answers can you find? Which answers are most reasonable? Explain.

22. Take It Further Find as many examples as you can of three different 1-digit numbers that are all divisible by +2 and have a sum of +4.

23. Take It Further Find all the divisors of –36. Write a division equation each time. Do you think –36 is a square number? Justify your answer.

24. Take It Further The mean daily high temperature in Rankin Inlet, Nunavut, during one week in January was –20°C. What might the temperatures have been on each day of the week? How many different possible answers can you find? Explain.

Reflect

You have learned different strategies for dividing integers. Which strategy do you prefer to use? Justify your choice. Listen to a classmate who has a different choice.

2.5 Order of Operations with Integers

Focus Apply the order of operations with integers.

How many different ways can you evaluate this expression?
$9 \times 6 + 36 \div 4 - 1$

The expression can also be written as: $9(6) + \frac{36}{4} - 1$

To ensure everyone gets the same value, use the order of operations.

Recall the order of operations with whole numbers.

- Do the operations in brackets first.
- Multiply and divide, in order, from left to right.
- Add and subtract, in order, from left to right.

The same order of operations applies to all integers.

Investigate

Work in groups of 4.
Choose 5 different integers between –10 and +10.
Use any operations or brackets.
Find the expression that has the greatest integer value.
Find the expression that has the least integer value.

Reflect & Share

Trade expressions with another group of classmates.
Find the values of your classmates' expressions.
Check that you and your classmates get the same answers.

Connect

Since we use curved brackets to show an integer; for example, (–2), we use square brackets to group terms. For example, $[(+9) - (-2)] \times (-3)$

When an expression is written as a fraction, the fraction bar indicates division. The fraction bar also acts like a grouping symbol. That is, the operations in the numerator and the denominator must be done first before dividing the numerator by the denominator.

Example 1

Evaluate: $[(-6) + (-2)] \div (-4) + (-5)$

A Solution

$[(-6) + (-2)] \div (-4) + (-5)$ Do the operation in square brackets first.

$= (-8) \div (-4) + (-5)$ Divide.

$= (+2) + (-5)$ Add.

$= -3$

Example 2

Evaluate: $\dfrac{2 + 4 \times (-8)}{-6}$

A Solution

$\dfrac{2 + 4 \times (-8)}{-6}$ Evaluate the numerator.
Multiply.

$= \dfrac{2 + (-32)}{-6}$ Add.

$= \dfrac{-30}{-6}$ Divide.

$= 5$

If an integer does not have a sign, it is assumed to be positive; for example, 2 = +2. Then we do not need to put the number in brackets.

Example 3

Evaluate: $\dfrac{[18 - (-6)] \times (-2)}{3(-4)}$

A Solution

$\dfrac{[18 - (-6)] \times (-2)}{3(-4)}$ Evaluate the numerator and denominator separately.
Do the square brackets first.

$= \dfrac{24 \times (-2)}{3(-4)}$ Multiply.

$= \dfrac{-48}{-12}$ Divide.

$= 4$

Discuss the ideas

1. Why are the square brackets unnecessary in this expression?
$(-3) + [12 \div (-4)]$
2. In *Example 3*, why were the numerator and denominator evaluated separately?

Practice

Check

3. State which operation you do first.

a) $7 + (-1) \times (-3)$

b) $(-18) \div (-6) - (-4)$

c) $6 + (-4) - (-2)$

d) $(-2)[7 + (-5)]$

e) $(-3) \times (-4) \div (-1)$

f) $8 - 3 + (-4) \div (-1)$

4. Evaluate each expression in question 3. Show all steps.

5. Elijah evaluated this expression as shown.

$$\begin{aligned} 3 - (-5) + 8(-4) &= 3 - (-5) + (-32) \\ &= 3 - (-37) \\ &= 40 \end{aligned}$$

Is Elijah's solution correct? If your answer is yes, explain the steps Elijah took. If your answer is no, what error did Elijah make? What is the correct answer? Show your work.

6. a) Evaluate.

i) $12 \div (2 \times 3) - 2$

ii) $12 \div 2 \times (3 - 2)$

b) Why are the answers different? Explain.

Apply

7. Evaluate. State which operation you do first.

a) $7(4) - 5$

b) $6[2 + (-5)]$

c) $(-3) + 4(7)$

d) $(-6) + 4(-2)$

e) $15 \div [10 \div (-2)]$

f) $18 \div 2(-6)$

8. Evaluate. Show all steps.

a) $6(5 - 7) - 3$

b) $4 - [5 + (-11)]$

c) $[4 - (-8)] \div 6$

d) $8 - 66 \div (-11)$

e) $(-24) \div 12 + (-3)(-4)$

f) $6(-3) + (-8)(-4)$

9. Evaluate. Show all steps.

a) $\dfrac{(-7) \times 4 + 8}{4}$

b) $\dfrac{4 + (-36) \div 4}{-5}$

c) $\dfrac{-32}{(-6)(-2) - (-4)}$

d) $\dfrac{9}{(-3) + (-18) \div 3}$

10. Evaluate. Show all steps.

a) $\dfrac{4(-3) + 7(-4)}{5(-1)}$

b) $\dfrac{[19 - (-5)] \div (-3)}{2(-2)}$

c) $\dfrac{32 \div 4 - (-28) \div 7}{12 \div (-4)}$

d) $\dfrac{12 - 4(-6)}{[3 - (-3)] \times (-3)}$

11. **Assessment Focus** Robert, Brenna, and Christian got different answers for this problem: $(-40) - 2[(-8) \div 2]$
Robert's answer was –32, Christian's answer was –48, and Brenna's answer was 168.

a) Which student had the correct answer?

b) Show and explain how the other two students got their answers. What errors did they make?

12. Evaluate each expression. Then insert one pair of square brackets in each expression so it evaluates to –5.

a) $(-20) \div 2 - (-2)$

b) $(-21) + 6 \div 3$

c) $10 + 3 \times 2 - 7$

13. Keisha had $405 in her bank account. In one month, she made 4 withdrawals of $45 each. What is the balance in her account? Write an integer expression to represent this problem. Solve the problem. How did you decide which operations to use?

14. Use three –4s and any operations or brackets. Write an expression with a value of:

a) –12 **b)** –4 **c)** 0

d) –3 **e)** 5 **f)** 2

15. **Take It Further** The daily highest temperatures for one week in February were: –2°C, +5°C, –8°C, –4°C, –11°C, –10°C, –5°C
Find the mean highest temperature. How did you decide which operations to use?

16. **Take It Further** Write an expression for each statement. Evaluate each expression.

a) Divide the sum of –24 and 4 by –5.

b) Multiply the sum of –4 and 10 by –2.

c) Subtract 4 from –10, then divide by –2.

17. **Take It Further** Copy each equation. Replace each $\square$ with the correct sign $(+, -, \times, \div)$ to make each equation true.

a) $(-10) \square (-2) \square 1 = 21$

b) $(-5) \square (-2) \square 4 = 1$

c) $6 \square (-7) \square 2 = -44$

d) $(-2)(-2) \square 8 = -4$

Reflect

Suppose you evaluate an expression that has different operations.
How do you know where to begin? How do you know what to do next?
Make up an integer expression that has three operations.
Explain how you evaluate it.

Understanding the Problem

Have you ever tried to solve
a math problem you didn't understand?
The first step in solving a problem is understanding it.

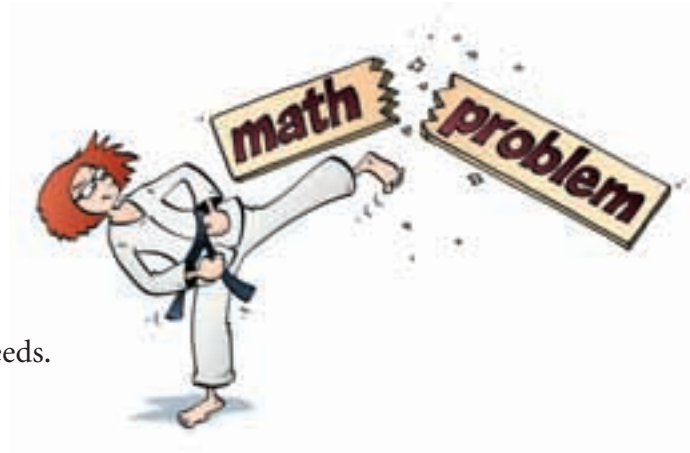
Consider this problem:
Work with a partner.
Use these integers: –9, –5, –2, 0, 1, 3, 5
Replace each * in the expression below
with an integer to get the greatest value.
Each integer can only be used once.

(*)(*) + (*) ÷ (*) − (*)

Explain why you placed the negative integers
where you did.
Why did this help produce the greatest value?

Suppose you don't understand a problem.
Here are some strategies you can use to help.

- Explain the problem to someone else in your own words.
- Break the problem down into parts.
  Read each part on its own.
  Think about what each part means.
- Highlight the important words.
- Decide what your answer will look like.
  Will your answer include:
  – a graph?
  – a number?
  – a table?
  – a diagram?
  – a written explanation?
- Think about how many parts your answer needs.



Strategies for Success

Here is one way to think about the problem:
Work with a partner.

Use these integers: –9, –5, –2, 0, 1, 3, 5
Replace each * in the expression below with an integer to get the greatest value.

These words help me make sense of what to do.

Each integer can only be used once.

(*) (*) + (*) ÷ (*) − (*)

Explain why you placed the negative integers where you did.
Why did this help produce the greatest value?

I should write a two-part answer.

Use at least one of the strategies on page 94 to help you understand each of these problems. Solve each problem.

1. Replace each * with a positive integer to make a true statement.
Find at least 4 different ways to do this.
(*) − (*) = −4

2. Insert one pair of square brackets in this expression so it has the least value. Explain your strategy.
(–5) + (+4) − (−2) ÷ (−1) × (−3)

3. Robbie collected these daily high temperatures for one week in January:
–10°C, –5°C, –1°C, 0°C, 1°C, 5°C
He forgot to record the temperature on the seventh day.
The newspaper reported that the mean high temperature for the week was 0°C.
What was the high temperature on the seventh day?
How do you know?

Unit Review

What Do I Need to Know?

Multiplying Integers

The product of two integers with the same sign is a positive integer.
$(+6) \times (+4) = +24$; $(-18) \times (-3) = +54$

The product of two integers with different signs is a negative integer.
$(-8) \times (+5) = -40$; $(+9) \times (-6) = -54$

The sign of a product with an even number of negative factors is positive.
$(-2) \times (-2) \times (-2) \times (-2) = +16$

The sign of a product with an odd number of negative factors is negative.
$(-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$

Dividing Integers

The quotient of two integers with the same sign is a positive integer.
$(+56) \div (+8) = \frac{+56}{+8} = +7$; $(-24) \div (-6) = \frac{-24}{-6} = +4$

The quotient of two integers with different signs is a negative integer.
$(-30) \div (+6) = \frac{-30}{+6} = -5$; $(+56) \div (-7) = \frac{+56}{-7} = -8$

Order of Operations

- Do the operations in brackets first.
- Multiply and divide, in order, from left to right.
- Add and subtract, in order, from left to right.

When the expression is written as a fraction:
- Evaluate the numerator and denominator separately.
- Then divide the numerator by the denominator.

What Should I Be Able to Do?

LESSON

2.1

1. Write each multiplication as a repeated addition. Then use coloured tiles to find each sum.

a) $(+2) \times (-1)$ **b)** $(+2) \times (+9)$

c) $(+3) \times (-3)$ **d)** $(+3) \times (+7)$

2. Use a model to find each product.

a) $(-7) \times (-5)$ **b)** $(+10) \times (-6)$

c) $(-4) \times (+4)$ **d)** $(+6) \times (+8)$

3. The temperature change in a chemistry experiment was –2°C every 30 min. The initial temperature was 6°C. What was the temperature after 4 h?

4. Will each product be positive or negative? How do you know?

a) $(+25) \times (-31)$ **b)** $(-13) \times (-15)$

c) $(-11) \times (+12)$ **d)** $(+9) \times (+13)$

2.2

5. Find each product.

a) $(+9) \times (-7)$ **b)** $(+4) \times (+7)$

c) $(-11) \times (+13)$ **d)** $(-40) \times (-22)$

e) $(-1) \times (+17)$ **f)** $(-37) \times 0$

6. Copy each equation. Replace $\square$ with an integer to make the equation true.

a) $(-12) \times \square = +72$

b) $\square \times (+8) = +80$

c) $(+7) \times \square = 0$

d) $\square \times (-4) = -60$

7. An old bucket has a small leak. Fifty-five millilitres of water leak out in 1 h. Use integers to find how much water leaks out in 6 h.

8. Write a word problem that could be solved using the expression $(+5) \times (-7)$. Solve the problem.

2.3

9. Use coloured tiles. Find each quotient. Sketch the tiles you used.

a) $(+15) \div (+3)$ **b)** $(+36) \div (-9)$

c) $(-21) \div (+7)$ **d)** $(-27) \div (-3)$

10. Use a model to find each quotient.

a) $(+18) \div (-3)$ **b)** $(+14) \div (+2)$

c) $(-28) \div (+4)$ **d)** $(-30) \div (-6)$

11. Tyler decides that, starting this week, he will withdraw \$5 from his savings account each week.

a) How many weeks from now will Tyler have withdrawn \$65?

b) Explain how you can use integers to model this situation.

c) What assumptions do you make?

2.4

12. Will each quotient be positive or negative? How do you know?

a) $(+26) \div (-2)$ **b)** $(-32) \div (-8)$

c) $(-1) \div (+1)$ **d)** $(+42) \div (+7)$

LESSON

13. Divide.

a) $(-56) \div (-7)$ **b)** $(+40) \div (-5)$

c) $(-88) \div (+8)$ **d)** $(+28) \div (+2)$

14. Divide.

a) $\frac{-18}{-2}$ **b)** $\frac{+16}{-4}$

c) $\frac{-18}{+6}$ **d)** $\frac{0}{-9}$

15. Divide.

a) $(+24) \div (-12)$ **b)** $(-63) \div (+21)$

c) $(+75) \div (+15)$ **d)** $(-99) \div (-11)$

16. Moira removed 3 candies from the jar every day. She now has removed 63 candies. For how many days did Moira remove candies?

a) Write this problem as a division expression using integers.

b) Solve the problem.

c) What assumptions do you make?

17. Write a word problem that could be solved using the expression $(+72) \div (-9)$. Solve the problem.

18. Find all the divisors of –21. Write a division equation each time.

2.5

19. State which operation you do first.

a) $4 - 6(-2)$

b) $(-18) \div (-9) - 3$

c) $[7 - (-3)] \div 5$

d) $4(-6) \div (-2)$

20. Evaluate each expression in question 19. Show all steps.

21. Evaluate.

a) $(-8) \div (-4) + 6(-3)$

b) $(-5) + (-12) \div (-3)$

c) $18 + 3[10 \div (-5)]$

d) $(-16) \div 8[7 - (-2)]$

22. Evaluate. Show all steps.

a) $\frac{3(-6) - 3}{-7}$

b) $\frac{(-4) + [(-7) - (-2)]}{3}$

c) $\frac{20}{(-3) + (-14) \div 7}$

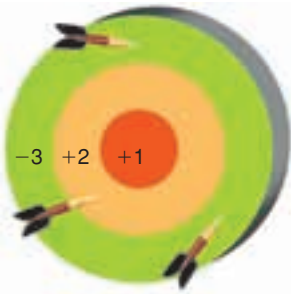
23. Evaluate. Show all steps.

a) $\frac{[18 - (-4)] \div (-11)}{(-4) + 2}$

b) $\frac{5(-2) + (-12) \div 3}{28 \div (-4)}$

c) $\frac{(-8)(-3)}{(-16) \div [(-13) - (-9)]}$

24. In a darts game, Suzanne and Corey each threw the darts 10 times. Corey had three (+2) scores, three (–3) scores, and four (+1) scores. Suzanne had four (+2) scores, four (–3) scores, and two (+1) scores.



a) What was each person's final score? How did you decide which operations to use?

b) The winner had the greater score. Who won the game? Explain.

Practice Test

1. Evaluate.

a) $(+9) \times (+10)$ **b)** $(+6) \times (-11)$ **c)** $(+96) \div (-16)$

d) $(+39) \div (+3)$ **e)** $(-8) \times (+6)$ **f)** $(-36) \div (+9)$

g) $(-44) \div (-4)$ **h)** $(-5) \times (-1)$

2. Evaluate.

a) $(-20)(-5) + 16 \div (-8)$ **b)** $\frac{14 - 10 \div 2}{-3}$

c) $\frac{[(-9) - (-2)] \times [8 + (-4)]}{(-14) \div (-2)}$ **d)** $[7 - (-2)] \times 2 + (-12) \div (-4)$

3. The temperature on Sunday was 4°C. The temperature dropped 8°C on Monday and dropped twice as much on Tuesday. What was the temperature on Tuesday? How did you decide which operations to use?

4. Suppose you own a store.
Use integers to model each situation.
For each situation, calculate the money you receive or spend.

a) Six people come into your store. Each person buys items worth $15.

b) You pay 3 bills. Each bill is for $35.

c) A supplier gives you $7 for each case of his product that you sell. You sold 9 cases this month.

5. Use the integers below.
0, –2, +3, –1, +1, +2, +4

a) Find two pairs of integers that have a quotient of –2.

b) Which two integers have the greatest product?

c) Which two integers have the least sum?

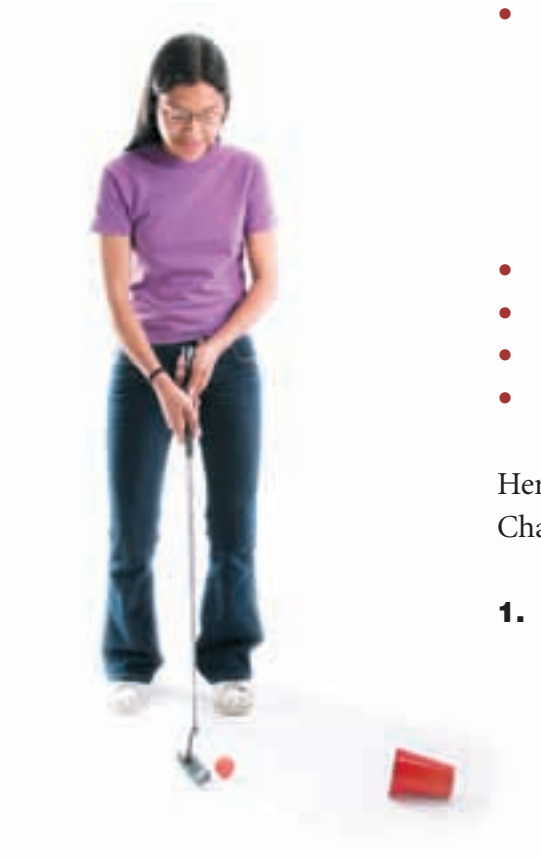
d) Which two integers have a quotient less than –3?

e) Write your own problem using two of the integers. Solve your problem. Show your work.

Unit Problem Charity Golf Tournament

A Grade 8 class and a local bank sponsor a golf tournament to raise money for local charities.
The bank provides these prizes:

1st place—$5000 to a charity of the player's choice
2nd and 3rd places—$1000 to a charity of the player's choice



Golf Terms

- "Par" is the number of strokes it should take for a player to reach the hole.
  If par is 3 and you take 5 strokes, then your score in relation to par is +2, or 2 over.
  If par is 3 and you take 2 strokes, then your score in relation to par is –1, or 1 under.
- A *bogey* is 1 stroke more than par, or 1 over par.
- A *double bogey* is 2 strokes more than par, or 2 over par.
- A *birdie* is 1 stroke less than par, or 1 under par.
- An *eagle* is 2 strokes less than par, or 2 under par.

Here are the top 6 golfers:
Chai Kim, Delaney, Hamid, Hanna, Kyle, and Weng Kwong

1. The golf course has 9 holes. Here is one person's results: par on 3 holes, a bogey on 2 holes, a birdie on 1 hole, an eagle on 2 holes, and a double bogey on 1 hole
   a) Write an integer expression to represent these results. How did you decide which operations to use?
   b) Evaluate the expression in part a to calculate the score in relation to par.

2. Chai Kim wrote his results in a table like this.

Hole	1	2	3	4	5	6	7	8	9
Par	3	4	3	3	5	4	4	3	3
Under/Over Par	0	–1	+2	0	–1	0	0	–1	0
Score	3	3	5	3					

a) Copy and complete the table. Use the following information:

Hole 1, 4, 6, 7, 9	Par
Hole 2, 5, 8	Birdie
Hole 3	Double bogey

b) What was Chai Kim's final score?

c) What was his final score in relation to par?

3. For each person below, make a table similar to the table in question 2. Use the information below. What is each golfer's final score?

a) Kyle:	Bogey holes	1, 3, 5, 9
	Birdie hole	6
	Par holes	2, 4, 7, 8
b) Delaney:	Bogey holes	3, 4, 6
	Birdie holes	1, 2, 7, 8, 9
	Eagle hole	5
c) Hamid:	Birdie every hole except hole 8	
	Double bogey hole 8	

4. **a)** Hanna had a score of –5 in relation to par.
Weng Kwong had a score of +3 in relation to par.
Use the information in questions 2 and 3.
Rank the players in order from least to greatest score.

b) Who won the tournament and the $5000 prize?
What was the score in relation to par?

c) Who won the $1000 prizes?
What were the scores in relation to par?

5. Use a table similar to that in question 2.
Complete the table with scores of your choice.
Calculate the final score, and the final score in relation to par.

Check List

Your work should show:

- ✓ how you used integers to solve the problems
- ✓ accurate calculations and ordering of integers
- ✓ tables you constructed to display the scores
- ✓ clear explanations, with correct use of mathematical language

Reflect on Your Learning

What did you find easy about working with integers? What was difficult for you? Give examples to illustrate your answers.