UNIT

2

8. **a)** Suppose you have 8 yellow tiles, and use all of them. How many red tiles would you need to model -3? How do you know?

b) Suppose you have 5 red tiles and 5 yellow tiles. How many ways can you find to model -3 with tiles?

9. Use coloured tiles to represent each sum. Find each sum. Sketch the tiles you used.

a) $(-7) + (+7)$ **b)** $(-7) + (+5)$

c) $(-7) + (-5)$ **d)** $(+7) + (-5)$

10. Use a number line.
For each sentence below:

a) Write each number as an integer.

b) Write an addition equation. Explain your answer in words.

i) You deposit \$10, then withdraw \$5.

ii) A balloon rises 25 m, then falls 10 m.

iii) You ride the elevator down 9 floors, then up 12 floors.

11. What is the difference in altitudes? How can you subtract to find out?

a) An altitude of 80 m above sea level and an altitude of 35 m below sea level

b) An altitude of 65 m below sea level and an altitude of 10 m above sea level

12. Add or subtract.

a) $(+5) + (-9)$ **b)** $(-1) + (-5)$

c) $(+2) - (-8)$ **d)** $(-9) - (-3)$

3

13. **a)** Write each fraction as a decimal.

i) $\frac{1}{33}$ **ii)** $\frac{2}{33}$ **iii)** $\frac{3}{33}$

b) Describe the pattern in your answers to part a.

c) Use your pattern to predict the fraction form of each decimal.

i) $0.\overline{15}$ **ii)** $0.\overline{24}$ **iii)** $0.\overline{30}$

14. **a)** Use any method. Order these numbers from greatest to least.
$\frac{21}{4}$, 4.9, $5\frac{1}{3}$, $\frac{24}{5}$, 5.3

b) Use a different method. Verify your answer in part a.

15. The tallest woman on record was 2.483 m tall. The shortest woman on record was 0.61 m tall. What is the difference in their heights?

16. Multiply. Draw a diagram to show each product.

a) 2.3×3.4 **b)** 1.8×2.2

c) 4.1×3.7 **d)** 1.7×2.9

17. Nuri has 10.875 L of water. He pours 0.5 L into each of several plastic bottles.

a) How many bottles can Nuri fill?

b) How much water is left over?

18. The Goods and Services Tax (GST) is currently 6%. For each item below:

i) Find the GST.

ii) Find the cost of the item including GST.

a) snowshoes that cost \$129.99

b) a CD that costs \$17.98

UNIT 4 Circles and Area

Look at these pictures.
What shapes do you see?

- What is a circle?
- Where do you see circles?
- What do you know about a circle?
- What might be useful to know about a circle?
  A parallelogram?
  A triangle?

What You'll Learn

- Investigate and explain the relationships among the radius, diameter, and circumference of a circle.
- Determine the sum of the central angles of a circle.
- Construct circles and solve problems involving circles.
- Develop formulas to find the areas of a parallelogram, a triangle, and a circle.
- Draw, label, and interpret circle graphs.

Why It's Important

- The ability to measure circles, triangles, and parallelograms is an important skill.
  These shapes are used in design, architecture, and construction.

Key Words

- radius, radii
- diameter
- circumference
- π
- irrational number
- base
- height
- circle graph
- sector
- legend
- percent circle
- central angle
- sector angle
- pie chart

4.1 Investigating Circles

Focus Measure radius and diameter and discover their relationship.

Explore

You will need circular objects, a compass, and a ruler.

➤ Use a compass. Draw a large circle. Use a ruler. Draw a line segment that joins two points on the circle. Measure the line segment. Label the line segment with its length. Draw and measure other segments that join two points on the circle. Find the longest segment in the circle. How many other segments can you draw with this length? Repeat the activity for other circles.

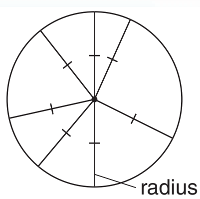
➤ Trace a circular object. Cut out the circle. How many ways can you find the centre of the circle? Measure the distance from the centre to the circle. Measure the distance across the circle, through its centre. Record the measurements in a table. Repeat the activity with other circular objects. What pattern do you see in your results?

Reflect & Share

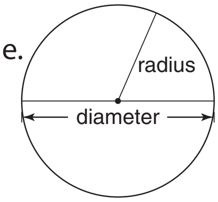
Compare your results with those of another pair of classmates.
Where is the longest segment in any circle?
What relationship did you find between the distance across a circle through its centre, and the distance from the centre to the circle?

Connect

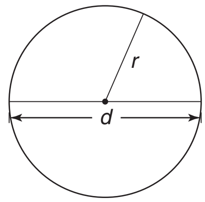
All points on a circle are the same distance from the centre of the circle. This distance is the **radius** of the circle.



The longest line segment in any circle is the **diameter** of the circle.
The diameter passes through the centre of the circle.
The radius is one-half the length of the diameter.
The diameter is two times the length of the radius.



Let r represent the radius, and d the diameter.
Then the relationship between the radius and diameter of a circle is:
$r = d \div 2$, which can be written as $r = \frac{d}{2}$
And, $d = 2r$



The plural of *radius* is *radii*; that is, one radius, two or more radii.

Example

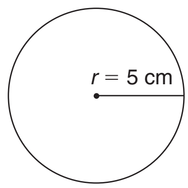
Use a compass. Construct a circle with:

a) radius 5 cm **b)** diameter 10 cm

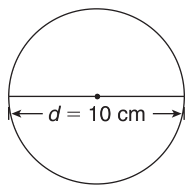
What do you notice about the circles you constructed?

A Solution

a) Draw a line segment with length 5 cm.
Place the compass point at one end.
Place the pencil point at the other end.
Draw a circle.



b) Draw a line segment with length 10 cm.
Use a ruler to find its midpoint.
Place the compass point at the midpoint.
Place the pencil point at one end of the segment.
Draw a circle.



The two circles are congruent.
A circle with radius 5 cm has diameter 10 cm.

Recall that congruent shapes are identical.

Practice

1. Use a compass.
Draw a circle with each radius.

a) 6 cm **b)** 8 cm

Label the radius, then find the diameter.

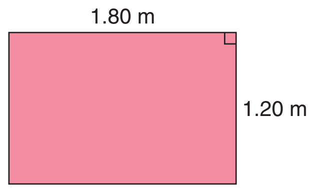
2. Draw a circle with each radius without using a compass.

a) 7 cm **b)** 4 cm

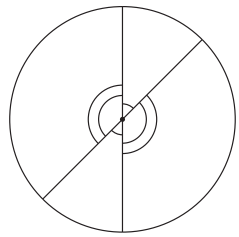
Label the radius, then find the diameter.
Explain the method you used to draw the circles.
What are the disadvantages of not using a compass?

3. **a)** A circle has diameter 3.8 cm. What is the radius?
b) A circle has radius 7.5 cm. What is the diameter?

4. A circular tabletop is to be cut from a rectangular piece of wood that measures 1.20 m by 1.80 m.
What is the radius of the largest tabletop that could be cut?
Justify your answer. Include a sketch.



5. **a)** Use a compass. Draw a circle. Draw 2 different diameters.
b) Use a protractor. Measure the angles at the centre of the circle.
c) Find the sum of the angles.
d) Repeat parts a to c for 3 different circles.
What do you notice about the sum of the angles in each circle?



6. A glass has a circular base with radius 3.5 cm.
A rectangular tray has dimensions 40 cm by 25 cm.
How many glasses will fit on the tray?
What assumptions did you make?

7. **Assessment Focus** Your teacher will give you a large copy of this logo.
Find the radius and diameter of each circle in this logo. Show your work.

This is the logo for the Aboriginal Health Department of the Vancouver Island Health Authority.

8. **Take It Further** A circular area of grass needs watering.
A rotating sprinkler is to be placed at the centre of the circle.
Explain how you would locate the centre of the circle.
Include a diagram in your explanation.

Reflect

How are the diameter and radius of a circle related?
Include examples in your explanation.

4.2 Circumference of a Circle

Focus Investigate the relationship between the circumference and diameter of a circle.

Explore

You will need 3 circular objects of different sizes, string, and a ruler.

➤ Each of you chooses one of the objects.
Use string to measure the distance around it.
Measure the radius and diameter of the object.
Record these measures.

➤ Repeat the activity until each of you has measured all 3 objects.
Compare your results.
If your measures are the same, record them in a table.
If your measures for any object are different, measure again to check.
When you agree upon the measures, record them in the table.

Object	Distance Around (cm)	Radius (cm)	Diameter (cm)
Can			

➤ What patterns do you see in the table?
How is the diameter related to the distance around?
How is the radius related to the distance around?

➤ For each object, calculate:
- distance around ÷ diameter
- distance around ÷ radius

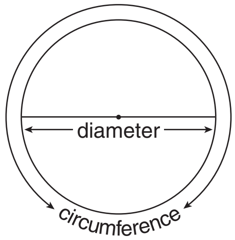
What do you notice?
Does the size of the circle affect your answers? Explain.

Reflect & Share

Compare your results with those of another group.
Suppose you know the distance around a circle.
How can you find its diameter?

Connect

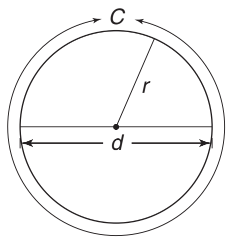
The distance around a circle is its **circumference**.
For any circle, the circumference, C, divided by the diameter, d, is approximately 3.
Circumference ÷ diameter $\doteq$ 3, or $\frac{C}{d} \doteq 3$



The circumference of a circle is also the perimeter of the circle.

For any circle, the ratio $\frac{C}{d} = \pi$
The symbol π is a Greek letter that we read as "pi."
$\pi = 3.141\,592\,653\,589\ldots$, or $\pi \doteq 3.14$

π is a decimal that never repeats and never terminates.
π cannot be written as a fraction.
For this reason, we call π an **irrational number**.



So, the circumference is π multiplied by d.
We write: $C = \pi d$

Since the diameter is twice the radius,
the circumference is also π multiplied by $2r$.
We write: $C = \pi \times 2r$, or $C = 2\pi r$

When we know the radius or diameter of a circle, we can use one of the formulas above to find the circumference of the circle.

The face of a toonie has radius 1.4 cm.

- To find the diameter of the face:
  The diameter $d = 2r$, where r is the radius
  Substitute: $r = 1.4$
  $d = 2 \times 1.4$
  $= 2.8$
  The diameter is 2.8 cm.

The circumference is a length, so its units are units of length such as centimetres, metres, or millimetres.

- To find the circumference of the face:

$C = \pi d$ OR $C = 2\pi r$

Substitute: $d = 2.8$ | Substitute: $r = 1.4$

$C = \pi \times 2.8$ | $C = 2 \times \pi \times 1.4$

$\doteq 8.796$ | $\doteq 8.796$

$\doteq 8.8$ | $\doteq 8.8$

The circumference is 8.8 cm, to one decimal place.

Use the π key on your calculator. If the calculator does not have a π key, use 3.14 instead.

- We can estimate to check if the answer is reasonable.
  The circumference is approximately 3 times the diameter:
  $3 \times 2.8 \text{ cm} \doteq 3 \times 3 \text{ cm}$
  $= 9 \text{ cm}$
  The circumference is approximately 9 cm.
  The calculated answer is 8.8 cm, so this answer is reasonable.

When we know the circumference, we can use a formula to find the diameter.
Use the formula $C = \pi d$.
To isolate d, divide each side by π.

$$\frac{C}{\pi} = \frac{\pi d}{\pi}$$

$$\frac{C}{\pi} = d$$

So, $d = \frac{C}{\pi}$

Example

An above-ground circular swimming pool has circumference 12 m.
Calculate the diameter and radius of the pool.
Give the answers to two decimal places.
Estimate to check the answers are reasonable.

A Solution

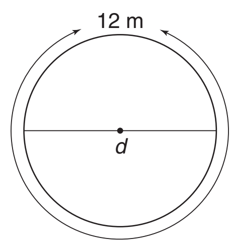
The diameter is: $d = \frac{C}{\pi}$

Substitute: $C = 12$

$d = \frac{12}{\pi}$

$= 3.8197\ldots$

Use a calculator.
Do not clear your calculator.

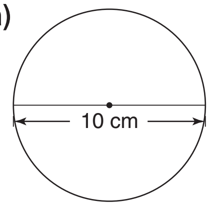


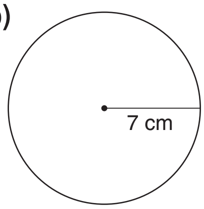
The radius is $\frac{1}{2}$ the diameter, or $r = d \div 2$.
Divide the number in the calculator display by 2.
$r \doteq 1.9099$
The diameter is 3.82 m to two decimal places.
The radius is 1.91 m to two decimal places.

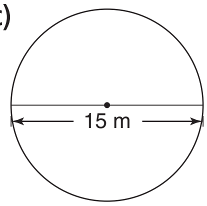
Since the circumference is approximately 3 times the diameter,
the diameter is about $\frac{1}{3}$ the circumference.
One-third of 12 m is 4 m. So, the diameter is about 4 m.
The radius is $\frac{1}{2}$ the diameter. One-half of 4 m is 2 m.
So, the radius of the pool is about 2 m.
Since the calculated answers are close to the estimates, the answers are reasonable.

Practice

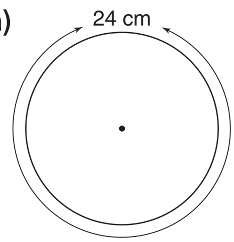
1. Calculate the circumference of each circle.
Give the answers to two decimal places.
Estimate to check the answers are reasonable.

a)


b)


c)


2. Calculate the diameter and radius of each circle.
Give the answers to two decimal places.
Estimate to check the answers are reasonable.

a)


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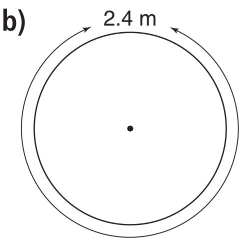
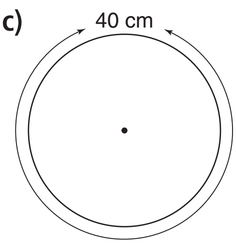


c)


3. When you estimate to check the circumference, you use 3 instead of π.
Is the estimated circumference greater than or less than
the actual circumference?
Why do you think so?

4. A circular garden has diameter 2.4 m.
a) The garden is to be enclosed with plastic edging.
How much edging is needed?
b) The edging costs $4.53/m.
What is the cost to edge the garden?

5. a) Suppose you double the diameter of a circle.
What happens to the circumference?
b) Suppose you triple the diameter of a circle.
What happens to the circumference?
Show your work.

6. A carpenter is making a circular tabletop with circumference 4.5 m.
What is the radius of the tabletop in centimetres?

Recall: 1 m = 100 cm

7. Can you draw a circle with circumference 33 cm?
If you can, draw the circle and explain how you know its circumference is correct.
If you cannot, explain why it is not possible.

8. **Assessment Focus** A bicycle tire has a spot of wet paint on it.
The radius of the tire is 46 cm.
Every time the wheel turns, the paint marks the ground.
a) What pattern will the paint make on the ground as the bicycle moves?
b) How far will the bicycle have travelled between two consecutive paint marks on the ground?
c) Assume the paint continues to mark the ground.
How many times will the paint mark the ground when the bicycle travels 1 km?
Show your work.

9. **Take It Further** Suppose a metal ring could be placed around Earth at the equator.
a) The radius of Earth is 6378.1 km. How long is the metal ring?
b) Suppose the length of the metal ring is increased by 1 km.
Would you be able to crawl under the ring, walk under the ring, or drive a school bus under the ring?
Explain how you know.

Reflect

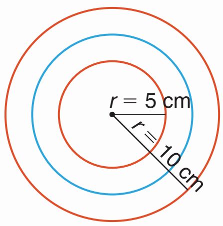
What is π?
How is it related to the circumference, diameter, and radius of a circle?

Mid-Unit Review

LESSON

4.1 **1.** **a)** Use a compass.
Draw a circle with radius 3 cm.
b) Do not use a compass.
Draw a circle with radius 7 cm.
The circle should have the same centre as the circle in part a.

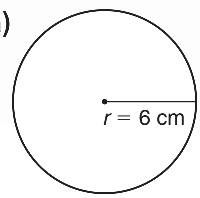
2. Two circles have the same centre. Their radii are 5 cm and 10 cm. Another circle lies between these circles. Give two possible diameters for this circle.

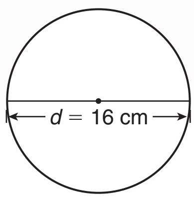


3. Find the radius of a circle with each diameter.
a) 7.8 cm **b)** 8.2 cm
c) 10 cm **d)** 25 cm

4. Is it possible to draw two different circles with the same radius and diameter? Why or why not?

4.2 **5.** Calculate the circumference of each circle. Give the answers to two decimal places. Estimate to check your answers are reasonable.

a)


b)


6. **a)** Calculate the circumference of each object.
i) A wheelchair wheel with diameter 66 cm
ii) A tire with radius 37 cm
iii) A hula-hoop with diameter 60 cm
b) Which object has the greatest circumference? How could you tell without calculating the circumference of each object?

7. Suppose the circumference of a circular pond is 76.6 m. What is its diameter?

8. Find the radius of a circle with each circumference. Give your answers to one decimal place.
a) 256 cm **b)** 113 cm **c)** 45 cm

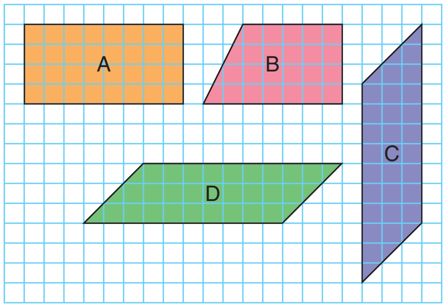
9. An auger is used to drill a hole in the ice, for ice fishing. The diameter of the hole is 25 cm. What is the circumference of the hole?

Area of a Parallelogram

Focus Develop a formula to find the area of a parallelogram.

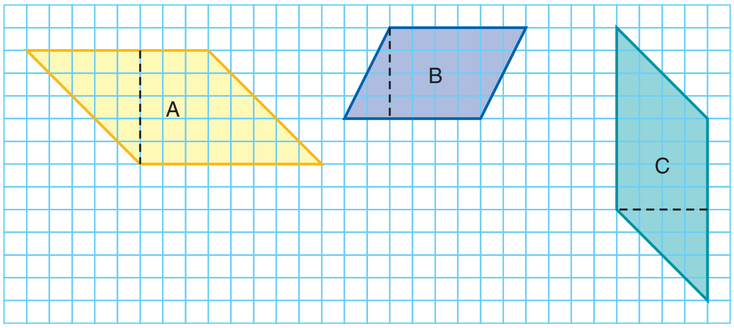
Which of these shapes are parallelograms?
How do you know?

How are Shapes C and D alike?
How are they different?



Explore

You will need scissors and 1-cm grid paper.



➤ Copy Parallelogram A on grid paper. Estimate, then find, the area of the parallelogram.

➤ Cut out the parallelogram. Then, cut along the broken line segment.

➤ Arrange the two pieces to form a rectangle.
What is the area of the rectangle?
How does the area of the rectangle compare to the area of the parallelogram?

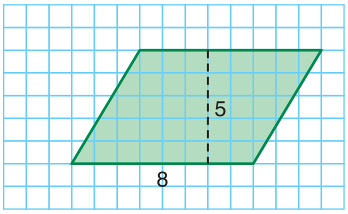
➤ Repeat the activity for Parallelograms B and C.

Reflect & Share

Share your work with another pair of classmates.
Can every parallelogram be changed into a rectangle by cutting and moving one piece? Explain.
Work together to write a rule for finding the area of a parallelogram.

Connect

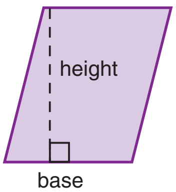
To estimate the area of this parallelogram, count the whole squares and the part squares that are one-half or greater.



There are:
- 33 whole squares
- 8 part squares that are one-half or greater

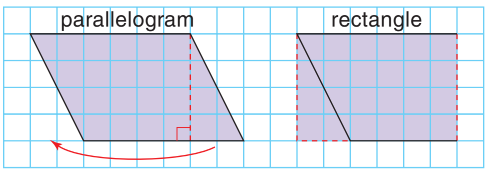
The area of this parallelogram is about 41 square units.

Any side of a parallelogram is a **base** of the parallelogram.
The **height** of a parallelogram is the length of a line segment that joins parallel sides and is perpendicular to the base.

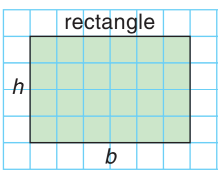


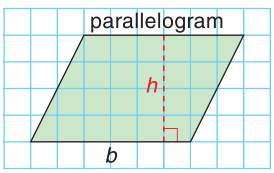
Recall that both a rectangle and a square are parallelograms.

Any parallelogram that is not a rectangle can be "cut" and rearranged to form a rectangle. Here is one way to do this.



The parallelogram and the rectangle have the same area.
The area of a parallelogram is equal to the area of a rectangle with the same height and base.
To find the area of a parallelogram, multiply the base by the height.





b represents the base.
h represents the height.

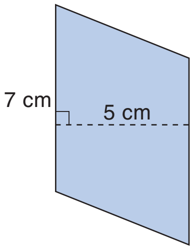
Area of rectangle:
$A = bh$

Area of parallelogram:
$A = bh$

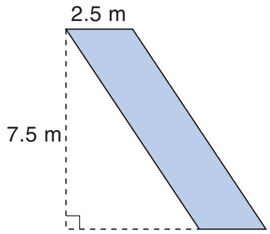
Example

Calculate the area of each parallelogram.

a)



b)



The height can be drawn outside the parallelogram.

A Solution

The area of a parallelogram is given by the formula $A = bh$.

a) $A = bh$

Substitute: $b = 7$ and $h = 5$

$A = 7 \times 5$

$= 35$

The area of the parallelogram is 35 cm^2.

b) $A = bh$

Substitute: $b = 2.5$ and $h = 7.5$

$A = 2.5 \times 7.5$

$= 18.75$

The area of the parallelogram is 18.75 m^2.

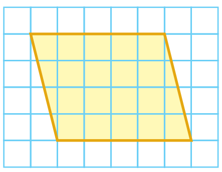
Practice

1. i) Copy each parallelogram on 1-cm grid paper.

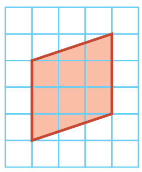
ii) Show how the parallelogram can be rearranged to form a rectangle.

iii) Estimate, then find, the area of each parallelogram.

a)



b)



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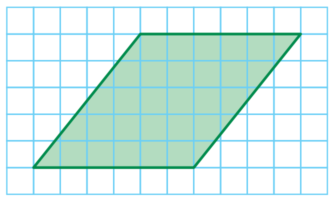
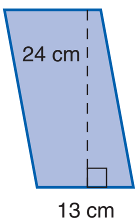


2. Find the area of each parallelogram.

a)



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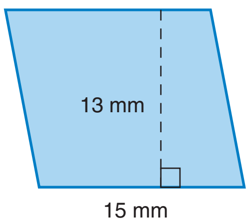

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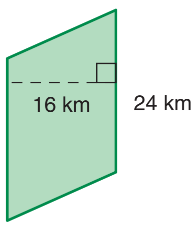
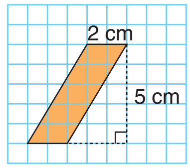


3. a) On 1-cm grid paper, draw 3 different parallelograms with base 3 cm and height 7 cm.

b) Find the area of each parallelogram you drew in part a. What do you notice?

4. Repeat question 3. This time, you choose the base and height. Are your conclusions the same as in question 3? Why or why not?

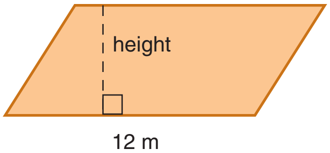
5. Copy this parallelogram on 1-cm grid paper.



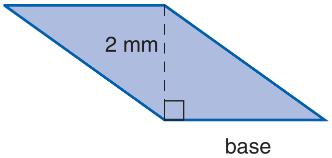
a) Show how this parallelogram could be rearranged to form a rectangle.
b) Find the area of the parallelogram.

6. Use the given area to find the base or the height of each parallelogram.

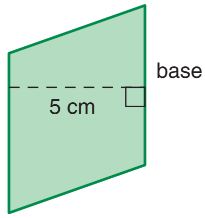
a) Area = 60 m^2



b) Area = 6 mm^2



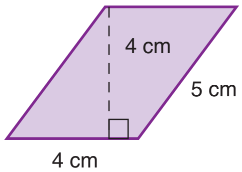
c) Area = 30 cm^2



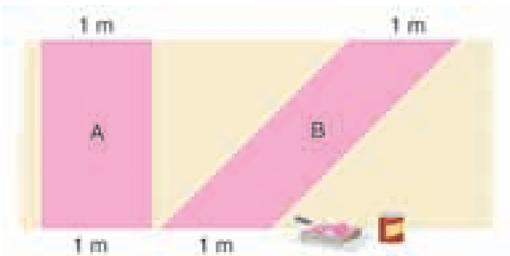
7. On 1-cm grid paper, draw as many different parallelograms as you can with each area.

a) 10 cm^2 b) 18 cm^2 c) 28 cm^2

8. A student says the area of this parallelogram is 20 cm^2. Explain the student's error.



9. **Assessment Focus** Sasha is buying paint for a design on a wall. Here is part of the design. Sasha says Shape B will need more paint than Shape A. Do you agree? Why or why not?



10. **Take It Further** A restaurant owner built a patio in front of his store to attract more customers.

a) What is the area of the patio?
b) What is the total area of the patio and gardens?
c) How can you find the area of the gardens?

Show your work.

Reflect

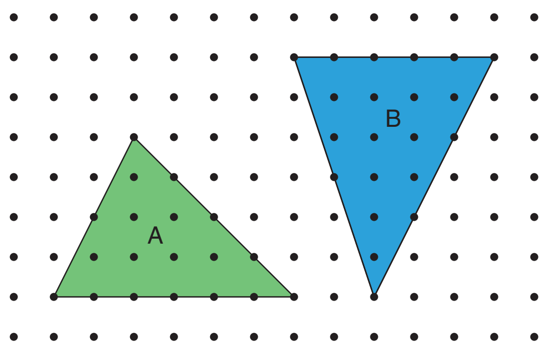
How can you use what you know about rectangles to help you find the area of a parallelogram? Use an example to explain.

4.4 Area of a Triangle

Focus Develop and use a formula to find the area of a triangle.

Explore

You will need a geoboard, geobands, and dot paper.



➤ Make Triangle A on a geoboard.
Add a second geoband to Triangle A to make a parallelogram with the same base and height.
This is called a *related* parallelogram.
Make as many different parallelograms as you can.
How does the area of the parallelogram compare to the area of Triangle A each time?
Record your work on dot paper.

➤ Repeat the activity with Triangle B.

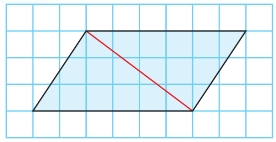
➤ What is the area of Triangle A? Triangle B?
What strategy did you use to find the areas?

Reflect & Share

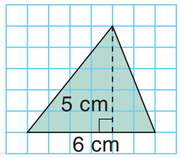
Share the different parallelograms you made with another pair of classmates.
Discuss the strategies you used to find the area of each triangle.
How did you use what you know about a parallelogram to find the area of a triangle?
Work together to write a rule for finding the area of a triangle.

Connect

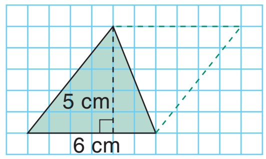
When we draw a diagonal in a parallelogram, we make two congruent triangles.
Congruent triangles have the same area.
The area of the two congruent triangles is equal to the area of the parallelogram that contains them.
So, the area of one triangle is $\frac{1}{2}$ the area of the parallelogram.



To find the area of this triangle:



Complete a parallelogram on one side of the triangle.
The area of the parallelogram is:
A = base $\times$ height, or $A = bh$
So, $A = 6 \times 5$
$\quad = 30$
The area of the parallelogram is 30 cm^2.
So, the area of the triangle is: $\frac{1}{2}$ of 30 cm^2 = 15 cm^2

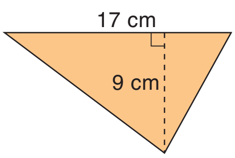


We can write a formula for the area of a triangle.
The area of a parallelogram is:
A = base $\times$ height
So, the area of a triangle is:
A = one-half of base $\times$ height
$A = bh \div 2$, which can be written as $A = \frac{bh}{2}$

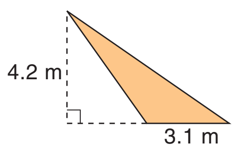
Example

Find the area of each triangle.

a)



b)



For an obtuse triangle, the height might be drawn outside the triangle.

A Solution

a) $A = \frac{bh}{2}$

Substitute: $b = 17$ and $h = 9$

$A = \frac{17 \times 9}{2}$

$= \frac{153}{2}$

$= 76.5$

The area is 76.5 cm^2.

b) $A = \frac{bh}{2}$

Substitute: $b = 3.1$ and $h = 4.2$

$A = \frac{3.1 \times 4.2}{2}$

$= \frac{13.02}{2}$

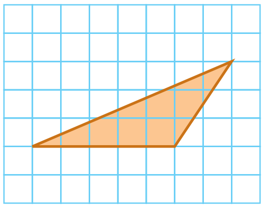
$= 6.51$

The area is 6.51 m^2.

Practice

1. Copy each triangle on 1-cm grid paper. Draw a related parallelogram.

a)



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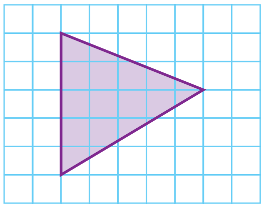

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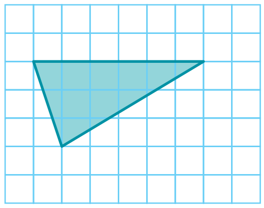
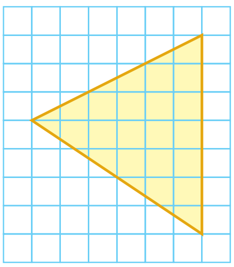
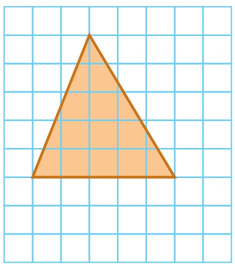


2. Each triangle is drawn on 1-cm grid paper.
Find the area of each triangle. Use a geoboard if you can.

a)



b)



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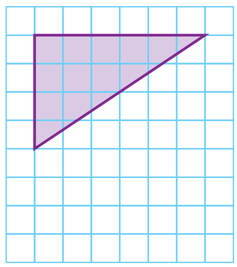

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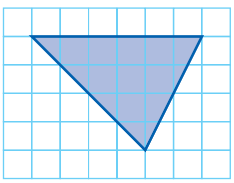
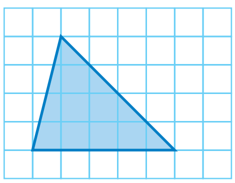
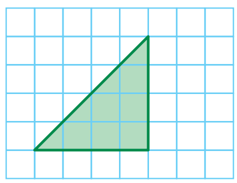


e)



f)



3. Draw two right triangles on 1-cm grid paper.

a) Record the base and the height of each triangle.

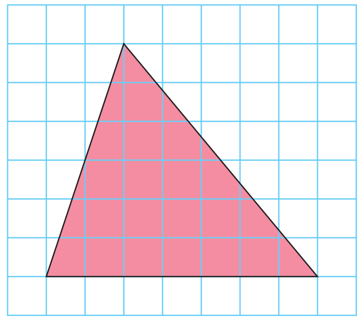
b) What do you notice about the height of a right triangle?

c) Find the area of each triangle you drew.

4. a) Find the area of this triangle.

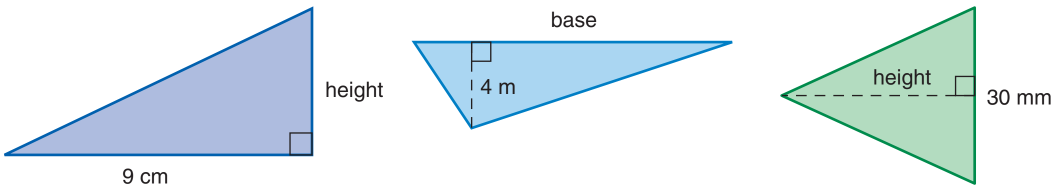
b) Use 1-cm grid paper.
How many different parallelograms can you draw that have the same base and the same height as this triangle?
Sketch each parallelogram.

c) Find the area of each parallelogram.
What do you notice?



5. Use the given area to find the base or height of each triangle.
How could you check your answers?

a) Area = 18 cm^2 b) Area = 32 m^2 c) Area = 480 mm^2



6. Use 1-cm grid paper.

a) Draw 3 different triangles with each base and height.
   i) base: 1 cm; height: 12 cm
   ii) base: 2 cm; height: 6 cm
   iii) base: 3 cm; height: 4 cm

b) Find the area of each triangle you drew in part a.
What do you notice?

7. On 1-cm grid paper, draw two different triangles with each area below.
Label the base and height each time.
How do you know these measures are correct?

a) 14 cm^2 b) 10 cm^2 c) 8 cm^2

8. a) Draw any triangle on grid paper.
What happens to the area of the triangle in each case?
   i) the base is doubled
   ii) both the height and the base are doubled
   iii) both the height and the base are tripled

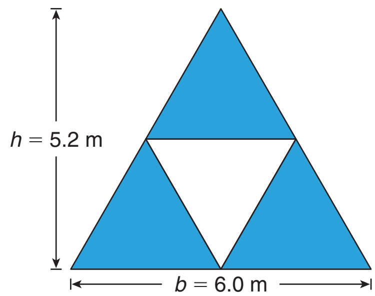
b) What could you do to the triangle you drew in part a to triple its area?
Explain why this would triple the area.

9. **Assessment Focus**

This triangle is made from 4 congruent triangles.
Three triangles are to be painted blue.
The fourth triangle is not to be painted.

a) What is the area that is to be painted?
Show your work.

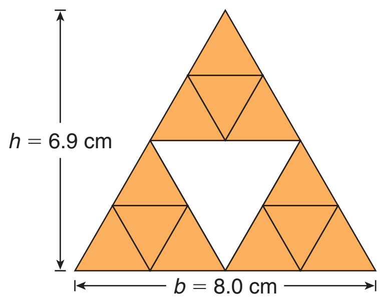
b) The paint is sold in 1-L cans.
One litre of paint covers 5.5 m^2.
How many cans of paint are needed?
What assumptions did you make?



The height is approximate.

10. Look at the diagram to the right.

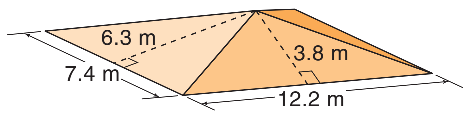
a) How many triangles do you see?
b) How are the triangles related?
c) How many parallelograms do you see?
d) Find the area of the large triangle.
e) Find the area of one medium-sized triangle.
f) Find the area of one small triangle.
g) Find the area of a parallelogram of your choice.

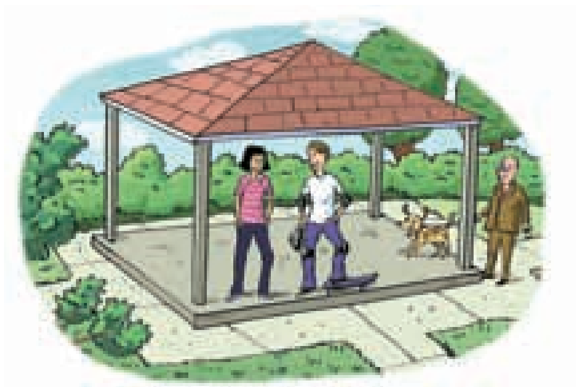


The height is approximate.

11. **Take It Further**

A local park has a pavilion to provide shelter.
The pavilion has a roof the shape of a rectangular pyramid.





a) What is the total area of all four parts of the roof?

b) One sheet of plywood is 240 cm by 120 cm.
What is the least number of sheets of plywood needed to cover the roof?
Explain how you got your answer.

Reflect

What do you know about finding the area of a triangle?

4.5 Area of a Circle

Focus Develop and use a formula to find the area of a circle.

Explore

You will need one set of fraction circles, masking tape, and a ruler.

➤ Each of you chooses one circle from the set of fraction circles. The circle you choose should have an even number of sectors, and at least 4 sectors.

➤ Each of you cuts 3 strips of masking tape:
- 2 short strips
- 1 strip at least 15 cm long

Use the short strips to fasten the long strip face up on the table.

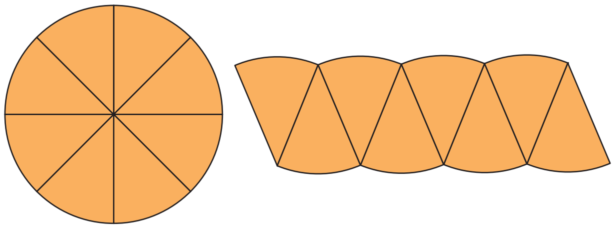
➤ Arrange all your circle sectors on the tape to approximate a parallelogram.
Trace your parallelogram, then use a ruler to make the horizontal sides straight.
Calculate the area of the parallelogram. Estimate the area of the circle.
How does the area of the parallelogram compare to the area of the circle?

Reflect & Share

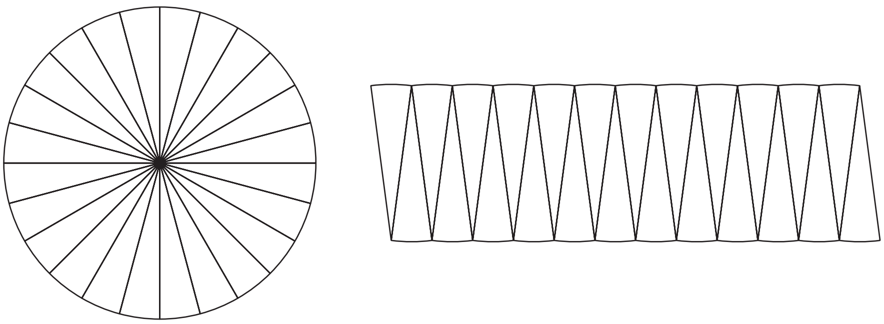
Compare your measure of the area of the circle with the measures of your group members.
Which area do you think is closest to the area of the circle? Why?
How could you improve your estimate for the area?
Which circle measure best represents the height of the parallelogram?
The base? Work together to write a formula for the area of a circle.

Connect

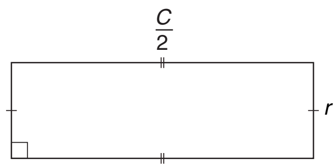
Suppose a circle was cut into 8 congruent sectors.
The 8 sectors were then arranged to approximate a parallelogram.



The more congruent sectors we use, the closer the area of the parallelogram is to the area of the circle.
Here is a circle cut into 24 congruent sectors.
The 24 sectors were then arranged to approximate a parallelogram.



The greater the number of sectors, the more the shape looks like a rectangle.
The sum of the two longer sides of the rectangle is equal to the circumference, C.
So, each longer side, or the base of the rectangle, is one-half the circumference of the circle, or $\frac{C}{2}$.

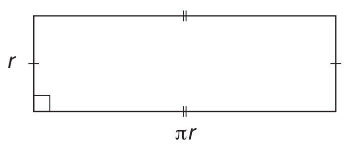


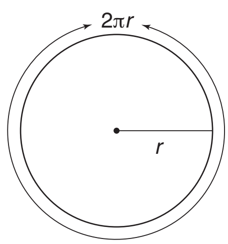
But $C = 2\pi r$

So, the base of the rectangle $= \frac{2\pi r}{2}$

$= \pi r$

Each of the two shorter sides is equal to the radius, r.

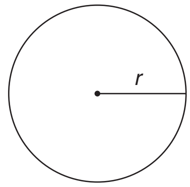




The area of a rectangle is: base $\times$ height
The base is πr. The height is r.
So, the area of the rectangle is: $\pi r \times r = \pi r^2$

When a number or variable is multiplied by itself we write: $7 \times 7 = 7^2$
$r \times r = r^2$

Since the rectangle is made from all sectors of the circle, the rectangle and the circle have the same area.
So, the area, A, of the circle with radius r is $A = \pi r^2$.



$A = \pi r^2$

We can use this formula to find the area of any circle when we know its radius.

Example

The face of a dime has diameter 1.8 cm.

a) Calculate the area.
Give the answer to two decimal places.

b) Estimate to check the answer is reasonable.

A Solution

The diameter of the face of a dime is 1.8 cm.


1.8 cm

So, its radius is: $\frac{1.8\text{ cm}}{2} = 0.9$ cm

a) Use the formula: $A = \pi r^2$
Substitute: $r = 0.9$
$A = \pi \times 0.9^2$
Use a calculator.
$A \doteq 2.544\ 69$
The area of the face of the dime is 2.54 cm^2 to two decimal places.

If your calculator does not have an x^2 key, key in 0.9×0.9 instead.

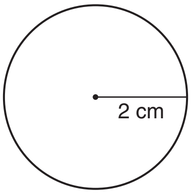
b) Recall that $\pi \doteq 3$.
So, the area of the face of the dime is about $3r^2$.

$$r \doteq 1$$
$$\text{So, } r^2 = 1$$
$$\text{and } 3r^2 = 3 \times 1$$
$$= 3$$

The area of the face of the dime is approximately 3 cm^2.
Since the calculated area, 2.54 cm^2, is close to 3 cm^2, the answer is reasonable.

Practice

1. Calculate the area of each circle.
Estimate to check your answers are reasonable.

a)


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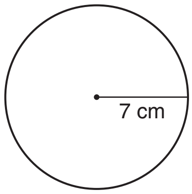

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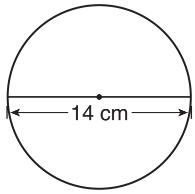

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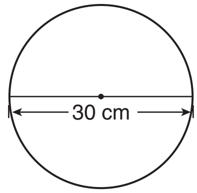
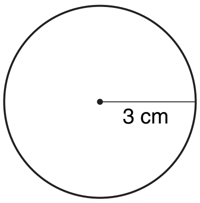
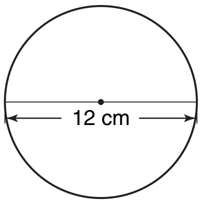


2. Calculate the area of each circle. Give your answers to two decimal places.
Estimate to check your answers are reasonable.

a)


b)


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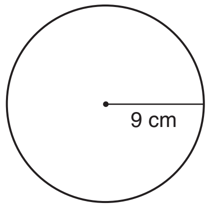

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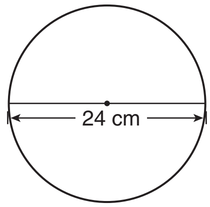
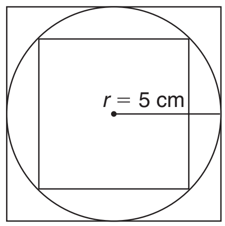


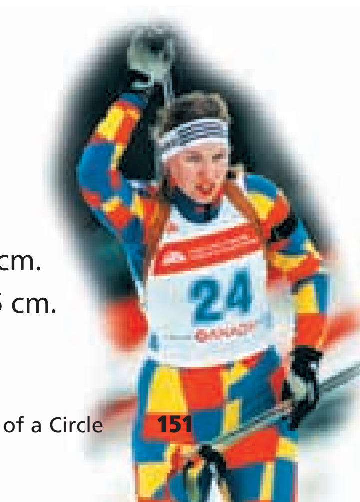
3. Use the results of questions 1 and 2. What happens to the area in each case?
a) You double the radius of a circle.
b) You triple the radius of a circle.
c) You quadruple the radius of a circle.
Justify your answers.

4. **Assessment Focus** Use 1-cm grid paper.
Draw a circle with radius 5 cm.
Draw a square outside the circle that just encloses the circle.
Draw a square inside the circle
so that its vertices lie on the circle.
Measure the sides of the squares.



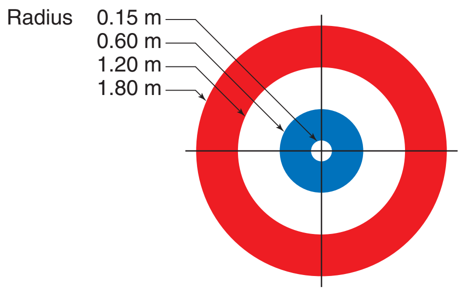
a) How can you use the areas of the two squares to estimate the area of the circle?
b) Check your estimate in part a by calculating the area of the circle.
c) Repeat the activity for circles with different radii.
Record your results. Show your work.

5. In the biathlon, athletes shoot at targets. Find the area of each target.
a) The target for the athlete who is standing is a circle with diameter 11.5 cm.
b) The target for the athlete who is lying down is a circle with diameter 4.5 cm.
Give the answers to the nearest square centimetre.

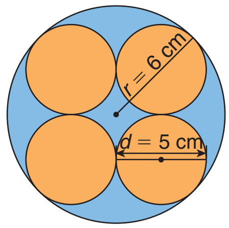


6. In curling, the target area is a bull's eye with 4 concentric circles.
   a) Calculate the area of the smallest circle.
   b) When a smaller circle overlaps a larger circle, a ring is formed.
      Calculate the area of each ring on the target area.
   Give your answers to 4 decimal places.

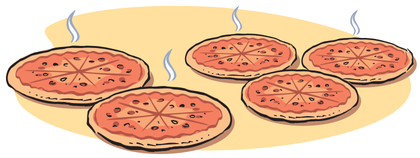
Concentric circles have the same centre.



7. **Take It Further**
   A circle with radius 6 cm contains 4 small circles.
   Each small circle has diameter 5 cm.
   Each small circle touches two other small circles and the large circle.
   a) Find the area of the large circle.
   b) Find the area of one small circle.
   c) Find the area of the region that is shaded yellow.



8. **Take It Further** A large pizza has diameter 35 cm.
   Two large pizzas cost $19.99.
   A medium pizza has diameter 30 cm.
   Three medium pizzas cost $24.99.
   Which is the better deal: 2 large pizzas or 3 medium pizzas?
   Justify your answer.



Math Link

Agriculture: Crop Circles
In Red Deer, Alberta, on September 17, 2001, a crop circle formation was discovered that contained 7 circles. The circle shown has diameter about 10 m. This circle destroyed some wheat crop. What area of wheat crop was lost in this crop circle?

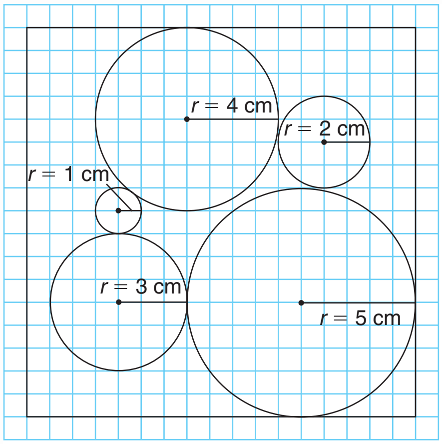
Reflect

You have learned two formulas for measurements of a circle.
How do you remember which formula to use for the area of a circle?

Packing Circles

Game

These circles are packed in a square.
In this game, you will pack circles in other shapes.



YOU WILL NEED

2 sheets of circles
scissors
ruler
compass
calculator

NUMBER OF PLAYERS

2

GOAL OF THE GAME

Construct the circle, triangle, and parallelogram with the lesser area.

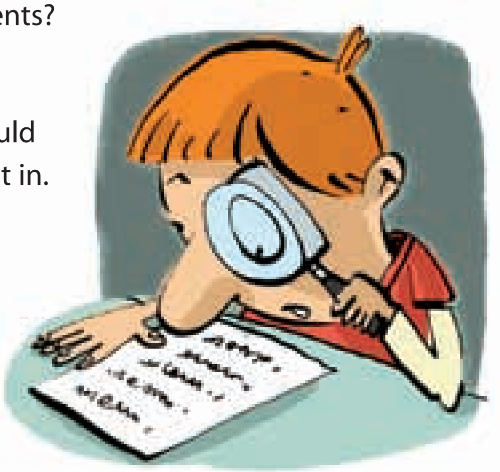
What strategies did you use to pack your circles to construct the shape with the lesser area?

HOW TO PLAY THE GAME:

1. Each player cuts out one sheet of circles.
2. Each player arranges his 5 circles so they are packed tightly together.
3. Use a compass. Draw a circle that encloses these circles.
4. Find the area of the enclosing circle.
   The player whose circle has the lesser area scores 2 points.
5. Pack the circles again.
   This time draw the parallelogram that encloses the circles.
   Find the area of the parallelogram.
   The player whose parallelogram has the lesser area scores 2 points.
6. Repeat *Step 5*. This time use a triangle to enclose the circles.
7. The player with the higher score wins.

Reading for Accuracy—Checking Your Work

Do you make careless mistakes on homework assignments? On quizzes or tests?



To help improve your accuracy and test scores, you should get in the habit of checking your work before handing it in.

The three most common types of mistakes are:

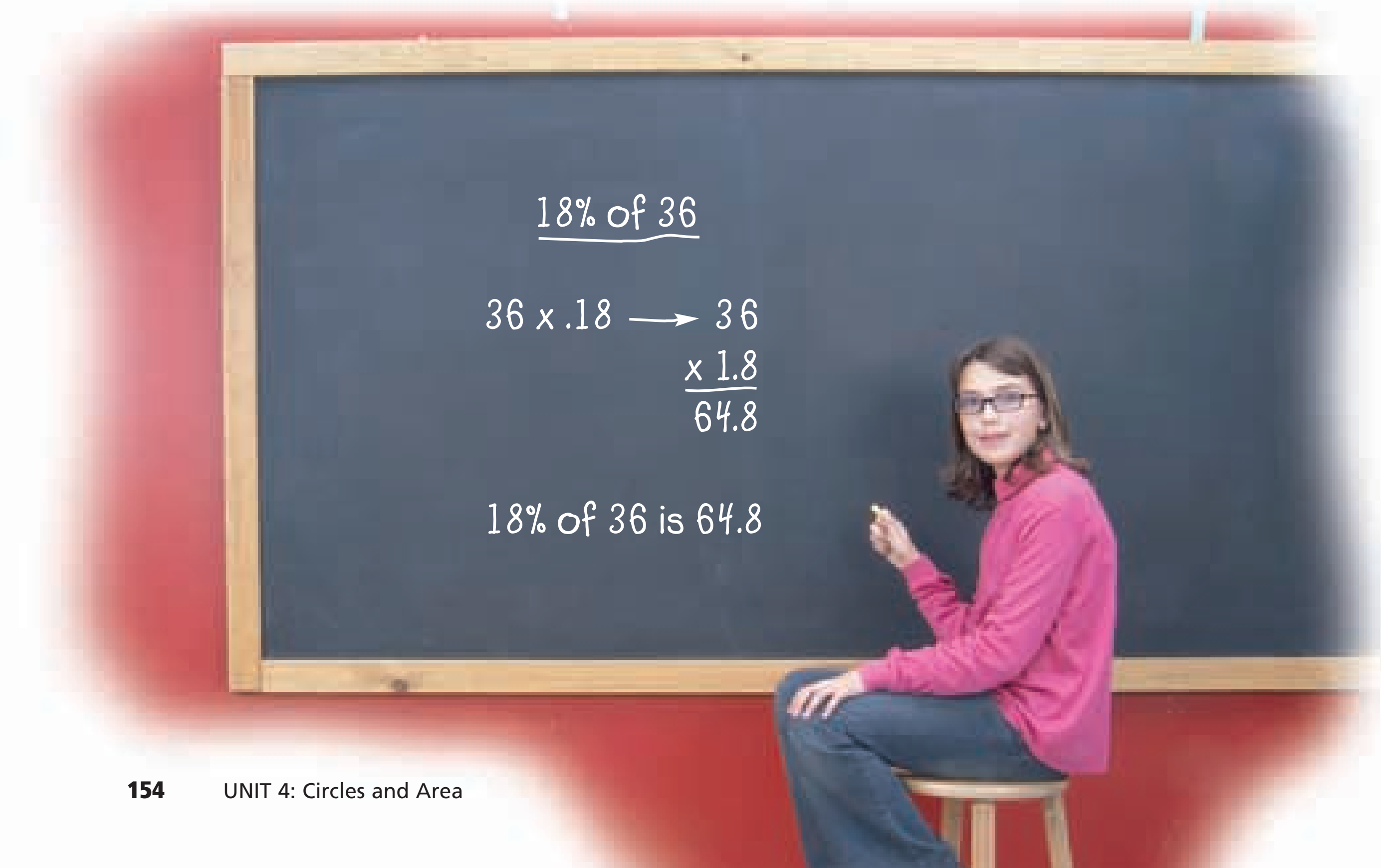
- copying errors
- notation errors
- calculation errors

Copying Errors

A copying error occurs when you copy the question or numbers in the question incorrectly.

Find the copying error in this solution.
How can you tell that the solution is not reasonable? Explain.

1. Find 18% of 36.



Notation Errors

A notation error occurs when you use a math symbol incorrectly.

Find the notation error in this solution.

2. Evaluate: (–8) + (+3) – (+2)

Evaluate $(-8) + (+3) - (+2)$

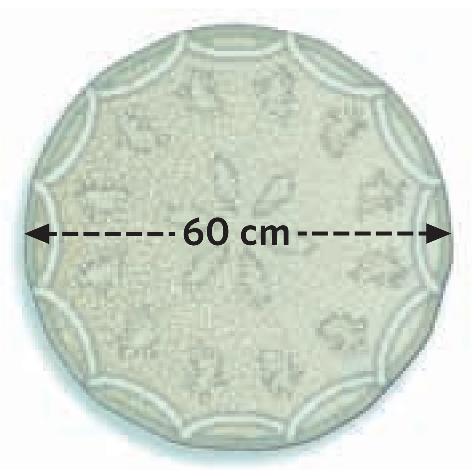
$$
\begin{aligned}
(-8) + (+3) - (+2) &= (-5) - (+2) \\
&= (+5) + (+2) \\
&= (+7)
\end{aligned}
$$

Calculation Errors

A calculation error occurs when you make a mistake in your calculations.

Find the calculation error in this solution.
How can you tell that the solution is not reasonable? Explain.

3. A circular mat has diameter 60 cm.
What is the area of the mat?



The diameter of the mat is 60 cm.
So, its radius is: 60 cm/2 = 30 cm
Use the formula $A = \pi r^2$
Substitute: $r = 30$

$$
\begin{aligned}
A &= \pi \times 30^2 \\
&= \pi \times 9000 \\
&\doteq 28\,274
\end{aligned}
$$

The area of the mat is about 28 274 cm².

4. Correct the error you found in each solution to find the correct answer.
Show your work.

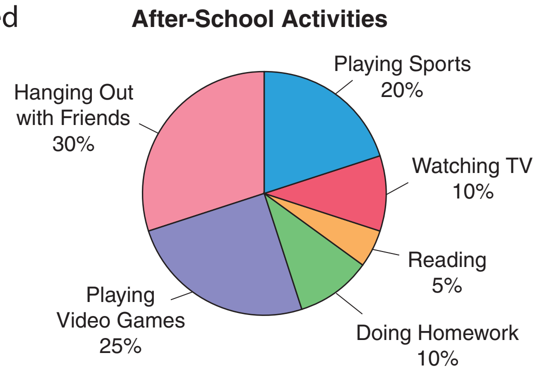
4.6 Interpreting Circle Graphs

Focus Interpret circle graphs to solve problems.

We can apply what we have learned about circles to interpret a new type of graph.

Explore

Sixty Grade 7 students at l'école Orléans were surveyed to find out their favourite after-school activity. The results are shown on the circle graph.



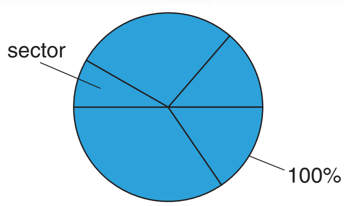
Which activity is most popular? Least popular?
How do you know this from looking at the graph?
How many students prefer each type of after-school activity? Which activity is the favourite for about $\frac{1}{3}$ of the students? Why do you think so?
Write 3 more things you know from looking at the graph.

Reflect & Share

Compare your answers with those of another pair of classmates.
What do you notice about the sum of the percents? Explain.

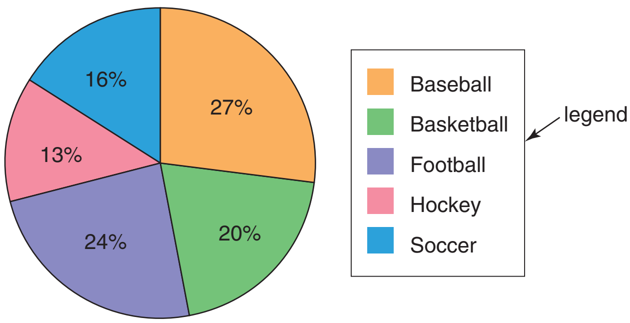
Connect

In a **circle graph**, data are shown as parts of one whole.
Each **sector** of a circle graph represents a percent of the whole circle.
The whole circle represents 100%.



A circle graph has a title.
Each sector is labelled with a category and a percent.
A circle graph compares the number in each category to the total number.
That is, a fraction of the circle represents the same fraction of the total.
Sometimes, a circle graph has a **legend** that shows what category each sector represents.
In this case, only the percents are shown on the graph.

Favourite Sports of Grade 7 Students



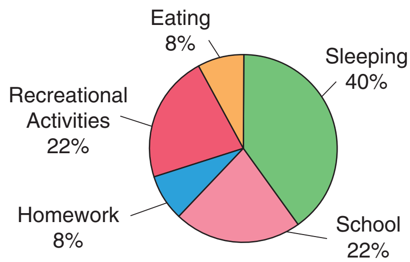
Example

This graph shows Nathan's typical day.

a) Which activity does Nathan do about $\frac{1}{4}$ of the time?

b) About how many hours does Nathan spend on each activity?
Check that the answers are reasonable.

Nathan's Typical Day



A Solution

a) Each of the sectors for "School" and "Recreational Activities" is about $\frac{1}{4}$ of the graph. 22% is close to 25%, which is $\frac{1}{4}$.
So, Nathan is in school about $\frac{1}{4}$ of the day.
He also participates in recreational activities about $\frac{1}{4}$ of the day.

b) ➤ From the circle graph, Nathan spends 40% of his day sleeping.
There are 24 h in a day.
Find 40% of 24.
$40\% = \frac{40}{100} = 0.4$
Multiply: $0.4 \times 24 = 9.6$
Nathan spends about 10 h sleeping.

9.6 is closer to 10 than to 9.

➤ Nathan spends 22% of his day in school.

Find 22% of 24.

$22\% = \frac{22}{100} = 0.22$

Multiply: $0.22 \times 24 = 5.28$

5.28 is closer to 5 than to 6.

Nathan spends about 5 h in school.

Nathan also spends about 5 h doing recreational activities.

➤ Nathan spends 8% of his day doing homework.

Find 8% of 24.

$8\% = \frac{8}{100} = 0.08$

Multiply: 0.08×24

Multiply as you would whole numbers.

$$\begin{array}{r} 24 \\ \times\ \ 8 \\ \hline 192 \end{array}$$

Estimate to place the decimal point.

$0.1 \times 24 = 2.4$

So, $0.08 \times 24 = 1.92$

1.92 is closer to 2 than to 1.

Nathan spends about 2 h doing homework.

Nathan also spends about 2 h eating.

The total number of hours spent on all activities should be 24, the number of hours in a day:

$9.6 + 5.28 + 5.28 + 1.92 + 1.92 = 24$

So, the answers are reasonable.

Add the exact times, *not* the approximate times.

Practice

1. This circle graph shows the most popular activities in a First Nations school. There are 500 students in the school. All students voted.

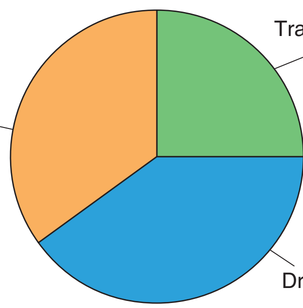
   a) Which activity did about $\frac{1}{4}$ of the students choose? How can you tell by looking at the graph?

   b) Which activity is the most popular? The least popular?

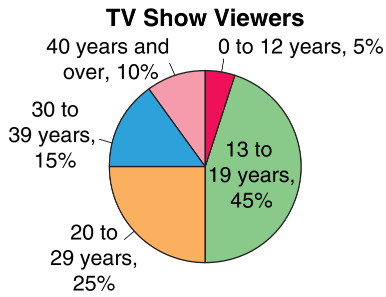
   c) Find the number of students who chose each activity.

   d) How can you check your answers to part c?

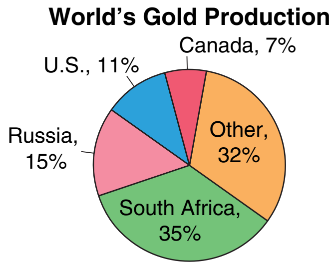
Favourite Activity at School



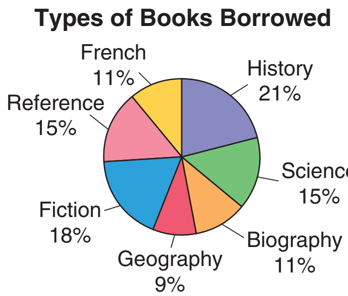
2. This circle graph shows the ages of viewers of a TV show.
One week, approximately 250 000 viewers tuned in.
a) Which two age groups together make up $\frac{1}{2}$ of the viewers?
b) How many viewers were in each age group?
i) 13 to 19 ii) 20 to 29 iii) 40 and over



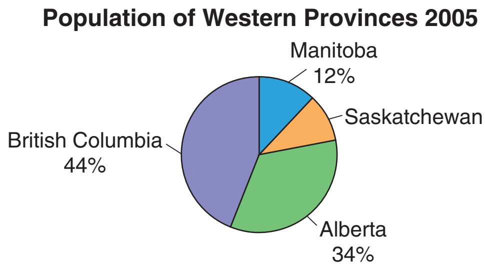
3. This graph shows the world's gold production for a particular year.
In this year, the world's gold production was approximately 2300 t.
About how much gold would have been produced in each country?
a) Canada b) South Africa



4. The school library budget to buy new books is $5000.
The librarian has this circle graph to show the types of books students borrowed in one year.
a) How much money should be spent on each type of book? How do you know?
b) Explain how you can check your answers in part a.



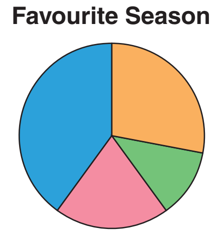
5. **Assessment Focus** This circle graph shows the populations of the 4 Western Canadian provinces in 2005.
The percent for Saskatchewan is not shown.
a) What percent of the population lived in Saskatchewan? How do you know?
b) List the provinces in order from least to greatest population.
How did the circle graph help you do this?
c) In 2005, the total population of the Western provinces was about 9 683 000 people.
Calculate the population of each province, to the nearest thousand.
d) What else do you know from looking at the circle graph? Write as much as you can.



6. Gaston collected data about the favourite season of his classmates.

Classmates' Favourite Season

Season	Autumn	Winter	Spring	Summer
Number of Students	7	3	5	10



He recorded the results in a circle graph.
The graph is not complete.

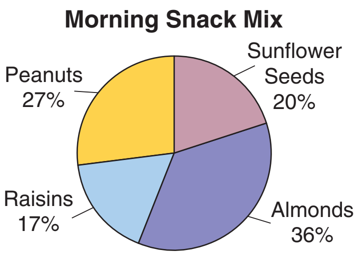
a) How many students were surveyed?

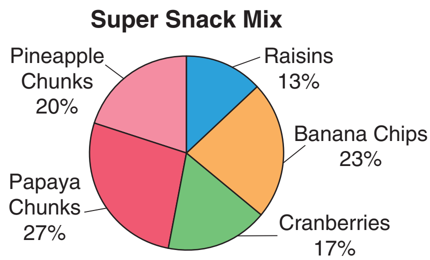
b) Write the number of students who chose each season as a fraction of the total number of students, then as a percent.

c) Explain how you can check your answers to part b.

d) Sketch the graph. Label each sector with its name and percent. How did you do this?

7. These circle graphs show the percent of ingredients in two 150-g samples of different snack mixes.





a) For each snack mix, calculate the mass, in grams, of each ingredient.

b) About what mass of raisins would you expect to find in a 300-g sample of each mix? What assumptions did you make?

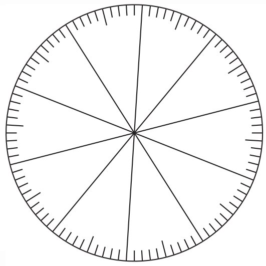
Reflect

Search newspapers, magazines, and the Internet to find examples of circle graphs.
Cut out or print the graphs.
How are they the same? How are they different?
Why were circle graphs used to display these data?

4.7 Drawing Circle Graphs

Focus Construct circle graphs to display data.

This is a **percent circle**.
The circle is divided into 100 congruent parts.
Each part is 1% of the whole circle.
You can draw a circle graph on a percent circle.



Explore

Your teacher will give you a percent circle.
Students in a Grade 7 class were asked how many siblings they have.
Here are the results.

0 Siblings	1 Sibling	2 Siblings	More than 2 Siblings
3	13	8	1

Write each number of students as a fraction of the total number.
Then write the fraction as a percent.

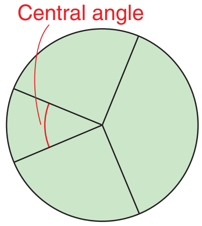
Use the percent circle.
Draw a circle graph to display the data.
Write 2 questions you can answer by looking at the graph.

Reflect & Share

Trade questions with another pair of classmates.
Use your graph to answer your classmates' questions.
Compare graphs. If they are different, try to find out why.
How did you use fractions and percents to draw a circle graph?

Connect

Recall that a circle graph shows how parts of a set of data compare with the whole set.
Each piece of data is written as a fraction of the whole.
Each fraction is then written as a percent.
Sectors of a percent circle are coloured to represent these percents.
The sum of the **central angles** is 360°.
A central angle is also called a **sector angle**.



Example

All the students in two Grade 7 classes were asked how they get to school each day. Here are the results: 9 rode their bikes, 11 walked, 17 rode the bus, and 13 were driven by car. Construct a circle graph to illustrate these data.

A Solution

- For each type of transport: Write the number of students as a fraction of 50, the total number of students. Then write each fraction as a decimal and as a percent.

  Bike: $\frac{9}{50} = \frac{18}{100} = 0.18 = 18\%$ Walk: $\frac{11}{50} = \frac{22}{100} = 0.22 = 22\%$

  Bus: $\frac{17}{50} = \frac{34}{100} = 0.34 = 34\%$ Car: $\frac{13}{50} = \frac{26}{100} = 0.26 = 26\%$

  The circle represents all the types of transport. To check, add the percents. The sum should be 100%.

  $18\% + 22\% + 34\% + 26\% = 100\%$

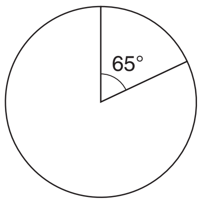
Another Strategy
We could use a percent circle to graph these data.

- To find the sector angle for each type of transport, multiply each decimal by 360°. Write each angle to the nearest degree, when necessary.

  Bike 18%: $0.18 \times 360° = 64.8° \doteq 65°$
  Walk 22%: $0.22 \times 360° = 79.2° \doteq 79°$
  Bus 34%: $0.34 \times 360° = 122.4° \doteq 122°$
  Car 26%: $0.26 \times 360° = 93.6° \doteq 94°$

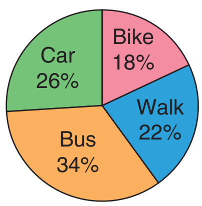
Check:
$64.8° + 79.2° + 122.4° + 93.6° = 360°$

- Construct a circle. Use a protractor to construct each sector angle. Start with the smallest angle. Draw a radius. Measure 65°.



  Start the next sector where the previous sector finished. Label each sector with its name and percent. Write a title for the graph.

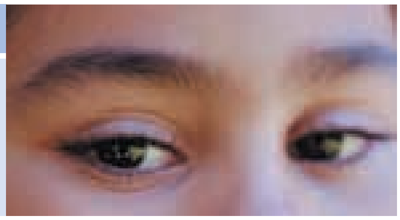
How Students Get to School



Practice

1. The table shows the number of Grade 7 students with each eye colour at Northern Public School.

Eye Colour	Number of Students
Blue	12
Brown	24
Green	8
Grey	6



a) Find the total number of students.
b) Write the number of students with each eye colour as a fraction of the total number of students.
c) Write each fraction as a percent.
d) Draw a circle graph to represent these data.

2. In a telephone survey, 400 people voted for their favourite radio station.
a) How many people chose EASY2?
b) Write the number of people who voted for each station as a fraction of the total number who voted. Then write each fraction as a percent.
c) Draw a circle graph to display the results of the survey.

Radio Station	Votes
MAJIC99	88
EASY2	?
ROCK1	120
HITS2	100

3. **Assessment Focus** This table shows the method of transport used by U.S. residents entering Canada in one year.
a) How many U.S. residents visited Canada that year?
b) What fraction of U.S. residents entered Canada by boat?
c) What percent of U.S. residents entered Canada by plane?
d) Display the data in a circle graph.
e) What else do you know from the table or circle graph? Write as much as you can.

United States Residents Entering Canada

Method of Transport	Number
Automobile	32 000 000
Plane	4 000 000
Train	400 000
Bus	1 600 000
Boat	1 200 000
Other	800 000

4. Can the data in each table below be displayed in a circle graph? Explain.

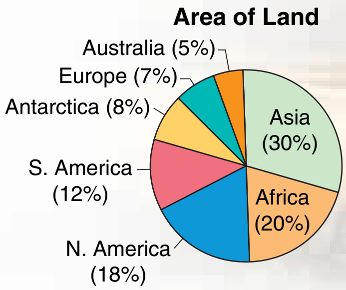
a)

Educational Attainment of Canadians	
0 to 8 years of elementary school	10%
Some secondary school	17%
Graduated from high school	20%
Some post-secondary education	9%
Post-secondary certificate or diploma	28%
University degree	16%

b)

Canadian Households with These Conveniences	
Automobile	64%
Cell phone	42%
Dishwasher	51%
Internet	42%

5. **Take It Further** This circle graph shows the percent of land occupied by each continent. The area of North America is approximately 220 million km^2.
Use the percents in the circle graph.
Find the approximate area of each of the other continents, to the nearest million square kilometres.



Reflect

When is it most appropriate to show data using a circle graph?
When is it not appropriate?

Using a Spreadsheet to Create Circle Graphs

Focus Display data on a circle graph using spreadsheets.

Spreadsheet software can be used to record, then graph, data. This table shows how Stacy budgets her money each month.

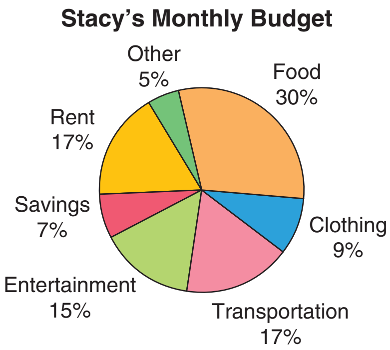
Stacy's Monthly Budget

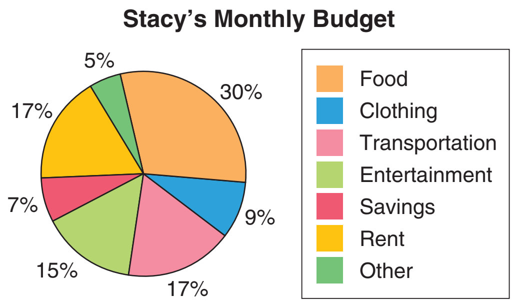
Category	Amount ($)
Food	160
Clothing	47
Transportation	92
Entertainment	78
Savings	35
Rent	87
Other	28

Enter the data into rows and columns of a spreadsheet. Highlight the data. Do not include the column heads.

	A	B
1	Category	Amount ($)
2	Food	160
3	Clothing	47
4	Transportation	92
5	Entertainment	78
6	Savings	35
7	Rent	87
8	Other	28

Click the graph/chart icon. In most spreadsheet programs, circle graphs are called **pie charts**. Select *pie chart*. Investigate different ways of labelling the graph. Your graph should look similar to one of the graphs on the following page.





**This circle graph shows a legend at the right.
The legend shows what category each sector represents.**

These data are from *Statistics Canada*.

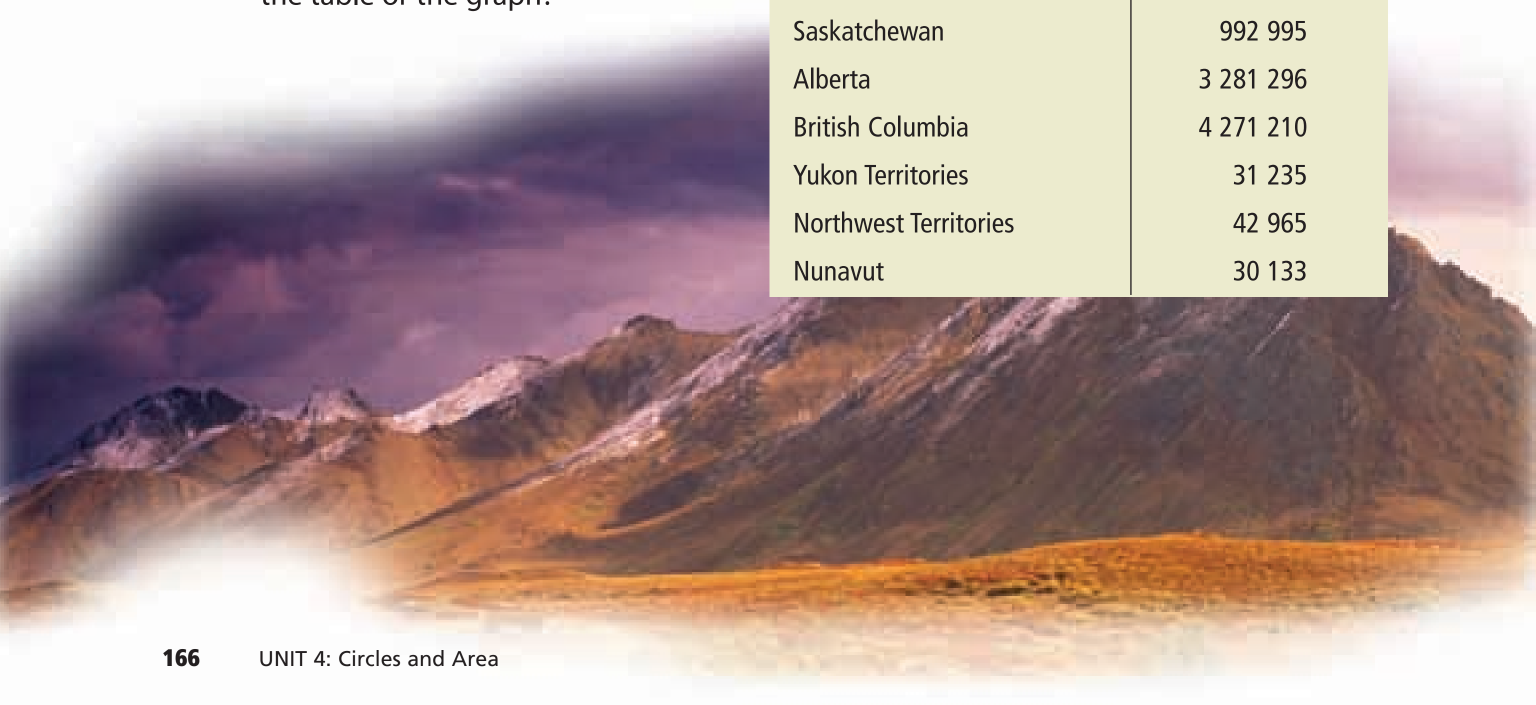
1. **a)** Use a spreadsheet.
Create a circle graph to display these data.

b) Write 3 questions about your graph.
Answer your questions.

c) Compare your questions with those of a classmate.
What else do you know from the table or the graph?

Population by Province and Territory, October 2005

Region	Population
Newfoundland and Labrador	515 591
Prince Edward Island	138 278
Nova Scotia	938 116
New Brunswick	751 726
Quebec	7 616 645
Ontario	12 589 823
Manitoba	1 178 109
Saskatchewan	992 995
Alberta	3 281 296
British Columbia	4 271 210
Yukon Territories	31 235
Northwest Territories	42 965
Nunavut	30 133

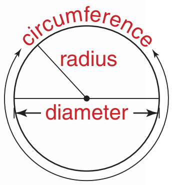


Unit Review

What Do I Need to Know?

Measurements in a Circle

The distance from the centre to a point on the circle is the *radius*. The distance across the circle, through the centre, is the *diameter*.
The distance around the circle is the *circumference*.



Circle Relationships

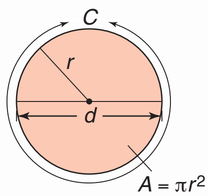
In a circle, let the radius be r, the diameter d, the circumference C, and the area A.

Then, $d = 2r$

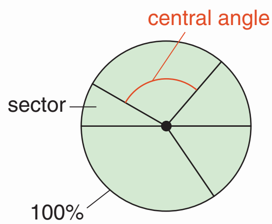
$$\frac{d}{2} = r$$

$$C = 2\pi r, \text{ or } C = \pi d$$

$$A = \pi r^2$$

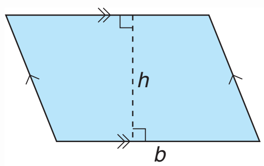


π is an irrational number that is approximately 3.14.
The sum of the central angles of a circle is 360°.

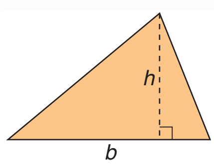


Area Formulas

Parallelogram: $A = bh$
where b is the base and
h is the height



Triangle: $A = \frac{bh}{2}$ or
$A = bh \div 2$
where b is the base and h is the height



Circle Graphs

In a circle graph, data are shown as parts of one whole.
The data are reported as a percent of the total, and the sum of the percents is 100%. The sum of the sector angles is 360°.

What Should I Be Able to Do?

LESSON

4.1

1. Draw a large circle without using a compass.
Explain how to find the radius and diameter of the circle you have drawn.

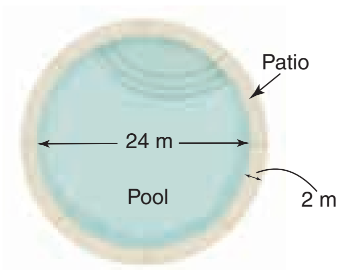
2. Find the radius of a circle with each diameter.
a) 12 cm **b)** 20 cm **c)** 7 cm

3. Find the diameter of a circle with each radius.
a) 15 cm **b)** 22 cm **c)** 4.2 cm

4.2

4. The circumference of a large crater is about 219 m.
What is the radius of the crater?

5. A circular pool has a circular concrete patio around it.
a) What is the circumference of the pool?
b) What is the combined radius of the pool and patio?
c) What is the circumference of the outside edge of the patio?



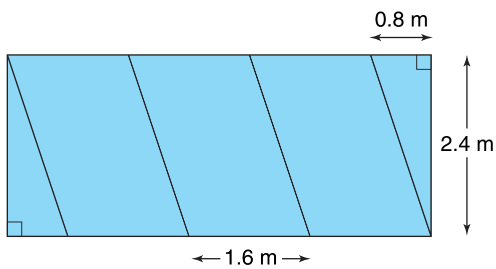
6. Mitra and Mel have different MP3 players.
The circular control dial on each player is a different size.
Calculate the circumference of the dial on each MP3 player.
a) Mitra's dial: diameter 30 mm
b) Mel's dial: radius 21 mm
c) Whose dial has the greater circumference? Explain.

4.3

7. On 0.5-cm grid paper, draw 3 different parallelograms with area 24 cm^2. What is the base and height of each parallelogram?

4.3
4.4

8. **a)** The window below consists of 5 pieces of glass. Each piece that is a parallelogram has base 1.6 m. What is the area of one parallelogram?



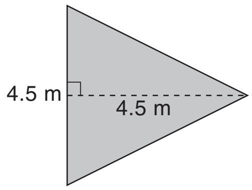
b) The base of each triangle in the window above is 0.8 m.
i) What is the area of one triangle?
ii) What is the area of the window?
Explain how you found the area.

9. On 0.5-cm grid paper, draw 3 different triangles with area 12 cm^2.

a) What is the base and height of each triangle?

b) How are the triangles related to the parallelograms in question 7?

10. Po Ling is planning to pour a concrete patio beside her house. It has the shape of a triangle. The contractor charges $125.00 for each square metre of patio.



What will the contractor charge for the patio?

4.5 **11.** A goat is tied to an 8-m rope in a field.

a) What area of the field can the goat graze?

b) What is the circumference of the area in part a?

12. Choose a radius. Draw a circle. Suppose you divide the radius by 2.

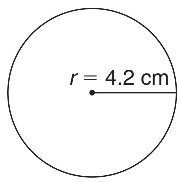
a) What happens to the circumference?

b) Explain what happens to the area.

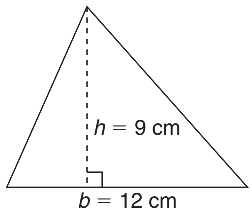
13. The diameter of a circular mirror is 28.5 cm. What is the area of the mirror? Give the answer to two decimal places.

14. Suppose you were to paint inside each shape below. Which shape would require the most paint? How did you find out?

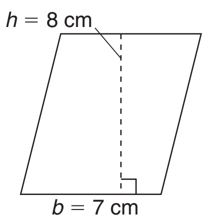
a)



b)



c)



LESSON

4.6

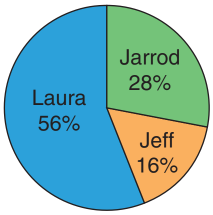
15. The results of the student council election are displayed on a circle graph. Five hundred students voted. The student with the most votes was named president.

a) Which student was named president? How do you know?

b) How many votes did each candidate receive?

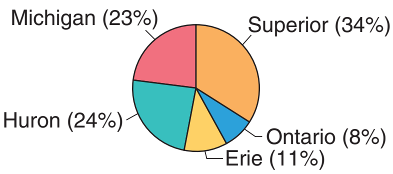
c) Write 2 other things you know from the graph.

Student Council Election Results



16. This circle graph shows the surface areas of the Great Lakes.

Areas of the Great Lakes



a) Which lake has a surface area about $\frac{1}{4}$ of the total area?

b) Explain why Lake Superior has that name.

c) The total area of the Great Lakes is about 244 000 km^2. Find the surface area of Lake Erie.

4.7

17. This table shows the approximate chemical and mineral composition of the human body.

Component	Percent
Water	62
Protein	17
Fat	15
Nitrogen	3
Calcium	2
Other	1

a) Draw a circle graph to display these data.

b) Jensen has mass 60 kg. About how many kilograms of Jensen's mass is water?

18. Here are the top 10 point scorers on the 2006 Canadian Women's Olympic Hockey Team. The table shows each player's province of birth.

Manitoba	Saskatchewan
Botterill	Wickenheiser
Quebec	**Ontario**
Ouellette	Apps
Goyette	Campbell
Vaillancourt	Hefford
	Piper
	Weatherston

a) What percent was born in each province?

b) Draw a circle graph to display the data in part a.

c) Why do you think more of these players come from Ontario than from any other province?

Practice Test

1. Draw a circle. Measure its radius.
Calculate its diameter, circumference, and area.

2. The circular frame of this dream catcher has diameter 10 cm.

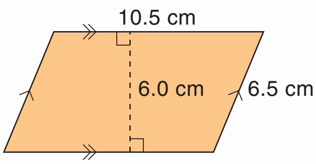
a) How much wire is needed to make the outside frame?

b) What is the area enclosed by the frame of this dream catcher?

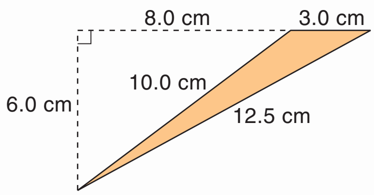
3. A circle is divided into 8 sectors.
What is the sum of the central angles of the circle? Justify your answer.

4. Find the area of each shape. Explain your strategy.

a)



b)



5. a) How many different triangles and parallelograms can you sketch with area 20 cm^2?
Have you sketched all possible shapes? Explain.

b) Can you draw a circle with area 20 cm^2?
If your answer is yes, explain how you would do it.
If your answer is no, explain why you cannot do it.

6. The table shows the type of land cover in Canada, as a percent of the total area.

a) Draw a circle graph.

b) Did you need to know the area of Canada to draw the circle graph? Explain.

c) Write 3 things you know from looking at the graph.

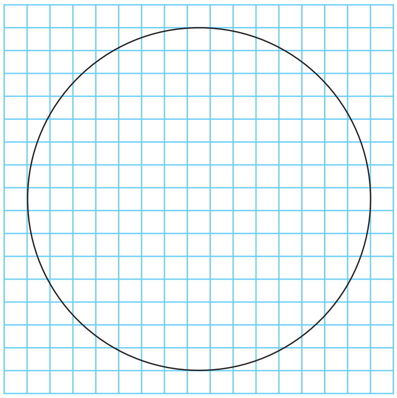
Type of Land Cover in Canada	
Forest and taiga	45%
Tundra	23%
Wetlands	12%
Fresh water	8%
Cropland and rangeland	8%
Ice and snow	3%
Human use	1%

Unit Problem Designing a Water Park

An anonymous donor gave a large sum of money to the Parks and Recreation Department.
The money is to be used to build a large circular water park.
Your task is to design the water park.

The water park has radius 30 m.

The side length of each square on this grid represents 4 m.



You must include the following features:

2 Wading Pools:

Each wading pool is triangular.
The pools do not have the same dimensions.
Each pool has area 24 m^2.

3 Geysers:

A geyser is circular.
Each geyser sprays water out of the ground,
and soaks a circular area with diameter 5 m or 10 m.

2 Wet Barricades:

A barricade has the shape of a parallelogram.
A row of nozzles in the barricade shoots water vertically.
The water falls within the area of the barricade.

4 Time-out Benches:

Each bench is shaped like a parallelogram.
It must be in the park.

At Least 1 Special Feature:

This feature will distinguish your park from other parks.
This feature can be a combination of any of the shapes you learned in this unit.
Give the dimensions of each special feature.
Explain why you included each feature in the park.

Check List

Your work should show:

- ✓ the area of each different shape you used
- ✓ a diagram of your design on grid paper
- ✓ an explanation of how you created the design
- ✓ how you calculated the area of the park that gets wet

Your teacher will give you a grid to draw your design.
You may use plastic shapes or cutouts to help you plan your park.
Complete the design.
Colour the design to show the different features.

Design your park so that a person can walk through the middle of the park without getting wet.
What area of the park will get wet?

Reflect on Your Learning

You have learned to measure different shapes.
When do you think you might use this knowledge outside the classroom?

Investigation

Digital Roots

Materials:
- multiplication chart
- compass
- protractor
- ruler

Work with a partner.

The **digital root** of a number is the result of adding the digits of the number until a single-digit number is reached.

For example, the digital root of 27 is: $2 + 7 = 9$
To find the digital root of 168:
Add the digits: $1 + 6 + 8 = 15$
Since 15 is not a single-digit number, add the digits: $1 + 5 = 6$
Since 6 is a single-digit number, the digital root of 168 is 6.

A digital root can also be found for the product of a multiplication fact.
For the multiplication fact, 8×4:
$8 \times 4 = 32$
Add the digits in the product: $3 + 2 = 5$
Since 5 is a single-digit number, the digital root of 8×4 is 5.

You will explore the digital roots of the products in a multiplication table, then display the patterns you find.
As you complete the *Investigation*, include all your work in a report that you will hand in.

Part 1

×	1	2	3	4	…
1					
2					
3					
4				7	
⋮					

➤ Use a blank 12×12 multiplication chart.
Find each product.
Find the digital root of each product.
Record each digital root in the chart.
For example, for the product $4 \times 4 = 16$, the digital root is $1 + 6 = 7$.

➤ Describe the patterns in the completed chart.
Did you need to calculate the digital root of each product?
Did you use patterns to help you complete the table?
Justify the method you used to complete the chart.

➤ Look down each column. What does each column represent?

Part 2

➤ Use a compass to draw 12 circles.
Use a protractor to mark 9 equally spaced points on each circle.
Label these points in order, clockwise, from 1 to 9.
Use the first circle.
Look at the first two digital roots in the 1st column of your chart.
Find these numbers on the circle.
Use a ruler to join these numbers with a line segment.
Continue to draw line segments to join points that match the digital roots in the 1st column.
What shape have you drawn?

➤ Repeat this activity for each remaining column.
Label each circle with the number at the top of the column.

➤ Which circles have the same shape?
Which circle has a unique shape?
What is unique about the shape?
Why do some columns have the same pattern of digital roots?
Explain.

Take It Further

➤ Investigate if similar patterns occur in each case:
- Digital roots of larger 2-digit numbers, such as 85 to 99
- Digital roots of 3-digit numbers, such as 255 to 269

Write a report on what you find out.

UNIT 5

Operations with Fractions

Many newspapers and magazines sell advertising space. Why would a company pay for an advertisement?

Selling advertising space is a good way to raise funds. Students at Anishwabe School plan to sell advertising space in their yearbook.

How can fractions be used in advertising space?

What You'll Learn

- Add and subtract fractions using models, pictures, and symbols.
- Add and subtract mixed numbers.
- Solve problems involving the addition and subtraction of fractions and mixed numbers.

Why It's Important

- You use fractions when you read gauges, shop, measure, and work with percents and decimals, and in sports, recipes, and business.

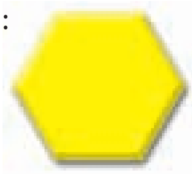
Key Words

- fraction strips
- simplest form
- related denominators
- unrelated denominators
- common denominator
- unit fraction

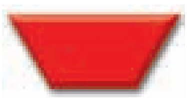
5.1 Using Models to Add Fractions

Focus Use Pattern Blocks and fraction circles to add fractions.

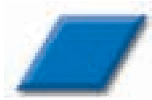
Let the yellow hexagon represent 1:



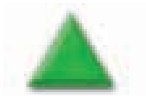
Then the red trapezoid represents $\frac{1}{2}$:



the blue rhombus represents $\frac{1}{3}$:



and the green triangle represents $\frac{1}{6}$:



Explore

Use Pattern Blocks.

Bakana trains for cross-country one hour a day. Here is her schedule:
Run for $\frac{1}{3}$ of the time, walk for $\frac{1}{6}$ of the time,
then run for the rest of the time.
How long does Bakana run altogether?
What fraction of the hour is this?

- Use fractions to write an addition equation to show how Bakana spent her hour.
- Bakana never runs for the whole hour.
  Write another possible schedule for Bakana.
  Write an addition equation for the schedule.
- Trade schedules with another pair of classmates.
  Write an addition equation for your classmates' schedule.

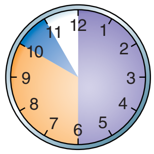
Reflect & Share

For the same schedule, compare equations with another pair of classmates.
Were the equations the same? How can you tell?
When are Pattern Blocks a good model for adding fractions?
When are Pattern Blocks not a good model?

Connect

There are many models that help us add fractions.

- We could use clocks to model halves, thirds, fourths, sixths, and twelfths.



$$\frac{1}{2} + \frac{1}{3} + \frac{1}{12} = \frac{11}{12}$$

Circle models are useful when the fractions are less than 1.

- The example below uses fraction circles to add fractions.

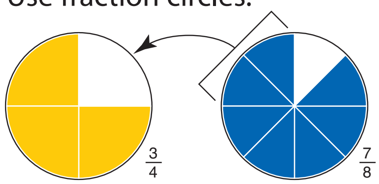
Example

Zack and Ronny each bought a small pizza.
Zack ate $\frac{3}{4}$ of his pizza and Ronny ate $\frac{7}{8}$ of his.
How much pizza did Zack and Ronny eat together?

A Solution

Add: $\frac{3}{4} + \frac{7}{8}$

Use fraction circles.



Use eighths to fill the circle for $\frac{3}{4}$.
Two-eighths fill the circle.

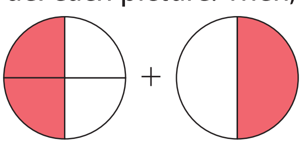
1 whole and 5 eighths equals $1\frac{5}{8}$.
So, $\frac{3}{4} + \frac{7}{8} = 1\frac{5}{8}$

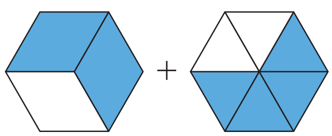
Together, Zack and Ronny ate $1\frac{5}{8}$ pizzas.

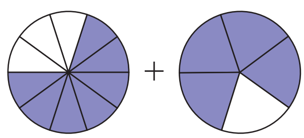
Practice

Use Pattern Blocks or fraction circles.

1. Model each picture. Then, find each sum.

a) 

b) 

c) 

2. Use a model to show each sum. Sketch the model.
Write an addition equation for each picture.

a) $\frac{7}{8} + \frac{1}{2}$ b) $\frac{3}{10} + \frac{2}{5}$ c) $\frac{2}{3} + \frac{1}{2}$ d) $\frac{2}{3} + \frac{5}{6}$

e) $\frac{3}{6} + \frac{1}{12}$ f) $\frac{1}{4} + \frac{2}{8}$ g) $\frac{1}{3} + \frac{1}{2}$ h) $\frac{1}{2} + \frac{4}{10}$

3. Simon spends $\frac{1}{6}$ h practising the whistle flute each day.
He also spends $\frac{1}{3}$ h practising the drums.
How much time does Simon spend each day practising these instruments?
Show how you found your solution.

4. a) Add.

i) $\frac{1}{5} + \frac{1}{5}$ ii) $\frac{2}{3} + \frac{1}{3}$ iii) $\frac{4}{10} + \frac{3}{10}$ iv) $\frac{1}{6} + \frac{3}{6}$

b) Look at your work in part a. How did you find your solutions?
How else could you add fractions with like denominators?

5. Is each sum greater than 1 or less than 1? How can you tell?

a) $\frac{1}{4} + \frac{2}{4}$ b) $\frac{2}{5} + \frac{7}{5}$ c) $\frac{3}{4} + \frac{1}{4}$ d) $\frac{1}{10} + \frac{3}{10}$

6. **Assessment Focus** Bella added 2 fractions. Their sum was $\frac{5}{6}$.
Which 2 fractions might Bella have added?
Find as many pairs of fractions as you can.
Show your work.

7. Asani's family had bannock with their dinner.
The bannock was cut into 8 equal pieces.
Asani ate 1 piece, her brother ate 2 pieces,
and her mother ate 3 pieces.

a) What fraction of the bannock did Asani eat?
Her brother? Her mother?

b) What fraction of the bannock was eaten?
What fraction was left?

Reflect

Which fractions can you add using Pattern Blocks? Fraction circles?
Give an example of fractions for which you cannot
use these models to add.

5.2 Using Other Models to Add Fractions

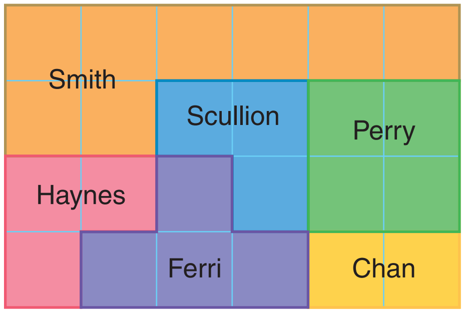
Focus Use fraction strips and number lines to add fractions.

We can use an area model to show fractions of one whole.

Explore

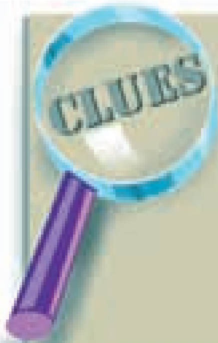
Your teacher will give you a copy of the map.
The map shows a section of land owned by 6 people.

- What fraction of land did each person own?
  What strategies did you use to find out?



Three people sold land to the other 3 people.

- Use the clues below to draw the new map.
- Write addition equations, such as $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$, to keep track of the land sales.



1. When all the sales were finished, four people owned all the land — Smith, Perry, Chan, and Haynes.
2. Smith now owns $\frac{1}{2}$ of the land.
3. Perry kept $\frac{1}{2}$ of her land, and sold the other half.
4. Chan bought land from two other people. He now owns $\frac{1}{4}$ of the land.
5. Haynes now owns the same amount of land as Perry started with.

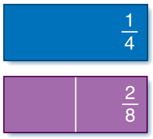
Reflect & Share

Did you find any equivalent fractions? How do you know they are equivalent?
Which clues helped you most to draw the new map? Explain how they helped.

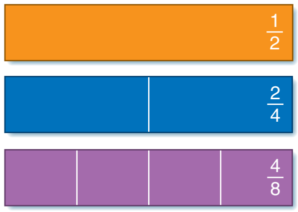
Connect

You can model fractions with strips of paper called **fraction strips**.

Here are more fraction strips and some equivalent fractions they show.



$\frac{1}{4} = \frac{2}{8}$

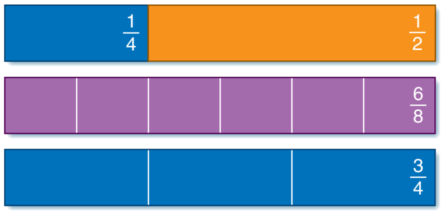


$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

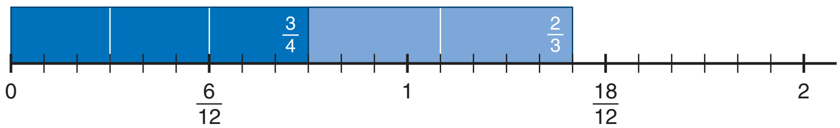
Recall that equivalent fractions show the same amount.

This strip represents 1 whole.

To add $\frac{1}{4} + \frac{1}{2}$, align the strips for $\frac{1}{4}$ and $\frac{1}{2}$.
Find a single strip that has the same length as the two strips.
There are 2 single strips: $\frac{6}{8}$ and $\frac{3}{4}$
So, $\frac{1}{4} + \frac{1}{2} = \frac{6}{8}$
And, $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$
$\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions.
The fraction $\frac{3}{4}$ is in simplest form.
A fraction is in **simplest form** when the numerator and denominator have no common factors other than 1.



When the sum is greater than 1, we could use fraction strips and a number line.



$\frac{3}{4} + \frac{2}{3} = \frac{17}{12}$

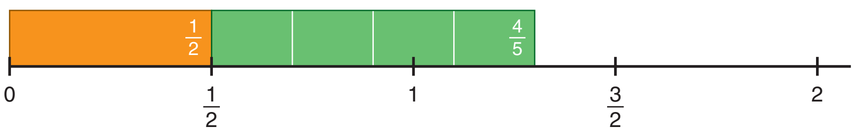
Example

Add. $\frac{1}{2} + \frac{4}{5}$

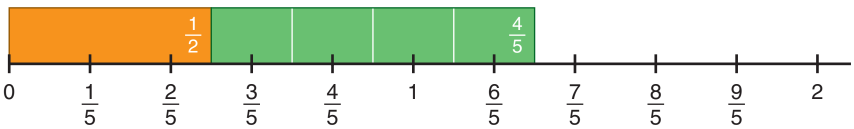
A Solution

$\frac{1}{2} + \frac{4}{5}$

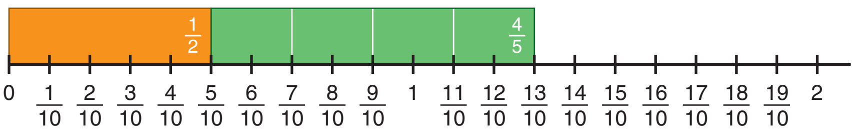
Place both strips end-to-end on the halves line.
The right end of the $\frac{4}{5}$-strip does not line up with a fraction on the halves line.



Place both strips on the fifths line.



The right end of the $\frac{4}{5}$-strip does not line up with a fraction on the fifths line.
Find a line on which to place both strips so the end of the $\frac{4}{5}$-strip lines up with a fraction.



Another Strategy
We could add these fractions using fraction circles.

The end of the $\frac{4}{5}$-strip lines up with a fraction on the tenths line.
The strips end at $\frac{13}{10}$. So, $\frac{1}{2} + \frac{4}{5} = \frac{13}{10}$

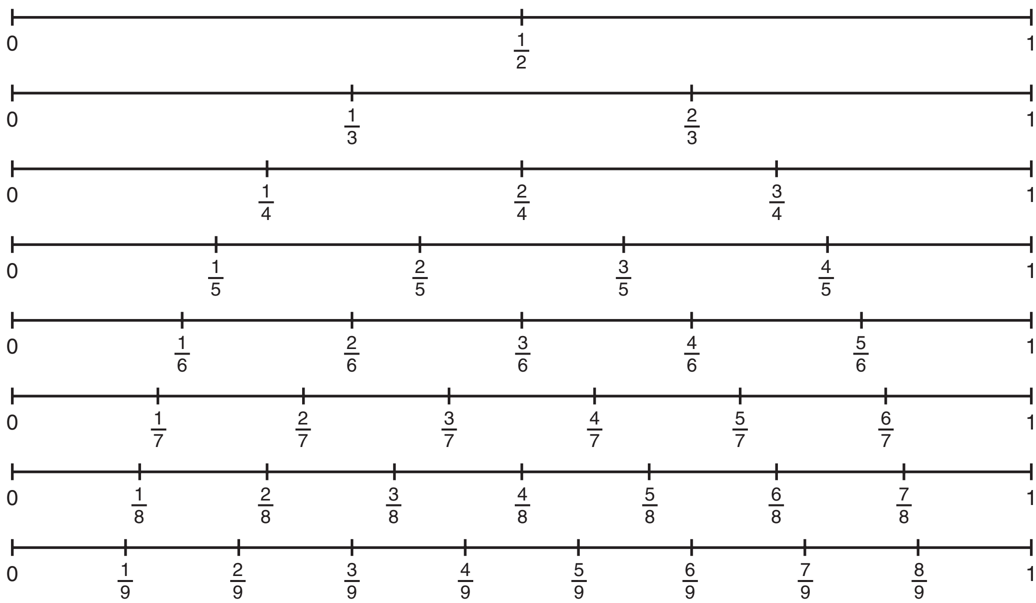
Practice

Use fraction strips and number lines.

1. Use the number lines below. List all fractions equivalent to:

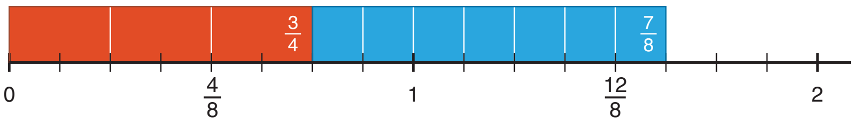
a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{2}{3}$

Use a ruler to align the fractions if it helps.

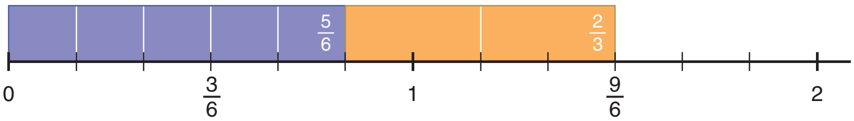


2. Write an addition equation for each picture.

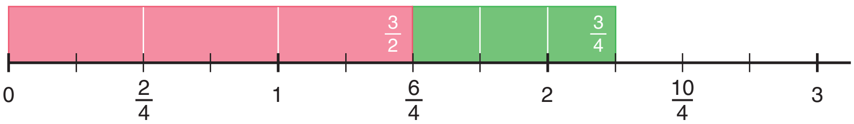
a)



b)



c)



3. Use your answers to question 2.

a) Look at the denominators in each part, and the number line you used to get the answer. What patterns do you see?

b) The denominators in each part of question 2 are **related denominators**. Why do you think they have this name?

4. Add.

a) $\frac{1}{3} + \frac{5}{6}$ b) $\frac{7}{12} + \frac{1}{3}$ c) $\frac{3}{5} + \frac{1}{10}$ d) $\frac{1}{6} + \frac{1}{12}$

5. Add.

a) $\frac{1}{3} + \frac{1}{2}$ b) $\frac{3}{4} + \frac{5}{6}$ c) $\frac{3}{5} + \frac{1}{2}$ d) $\frac{2}{3} + \frac{1}{5}$

6. Look at your answers to question 5.

a) Look at the denominators in each part, and the number line you used to get the answer. What patterns do you see?

b) The denominators in each part of question 5 are called **unrelated denominators**. Why do you think they have this name?

c) When you add 2 fractions with unrelated denominators, how do you decide which number line to use?

7. Add.

a) $\frac{1}{3} + \frac{2}{7}$ b) $\frac{3}{4} + \frac{2}{9}$ c) $\frac{4}{5} + \frac{5}{8}$ d) $\frac{2}{5} + \frac{3}{7}$

8. Abey and Anoki are eating chocolate bars.
The bars are the same size.
Abey has $\frac{3}{4}$ left. Anoki has $\frac{5}{6}$ left.
How much chocolate is left altogether? Show your work.

9. **Assessment Focus** Use any of the digits 1, 2, 3, 4, 5, 6 only once. Copy and complete. Replace each □ with a digit.

$$\frac{\square}{\square} + \frac{\square}{\square}$$

a) Find as many sums as you can that are between 1 and 2.
b) Find the least sum that is greater than 1.
Show your work.

10. Find 2 fractions with a sum of $\frac{3}{2}$. Try to do this as many ways as you can. Record each way you find.

11. **Take It Further** A jug holds 2 cups of liquid. A recipe for punch is $\frac{1}{2}$ cup of orange juice, $\frac{1}{4}$ cup of raspberry juice, $\frac{3}{8}$ cup of grapefruit juice, and $\frac{5}{8}$ cup of lemonade. Is the jug big enough for the punch? Explain how you know.

12. **Take It Further** A pitcher of juice is half empty. After $\frac{1}{2}$ cup of juice is added, the pitcher is $\frac{3}{4}$ full. How much juice does the pitcher hold when it is full? Show your thinking.

Math Link

Music
Musical notes are named for fractions. The type of note shows a musician how long to play the note. In math, two halves make a whole — in music, two half notes make a whole note!

whole note	half note	quarter note	eighth note	sixteenth note

Reflect

What do you now know about adding fractions that you did not know at the beginning of the lesson?

5.3 Using Symbols to Add Fractions

Focus Use common denominators to add fractions.

In Lessons 5.1 and 5.2, you used models to add fractions.
You may not always have suitable models.

You need a strategy you can use to add fractions without using a model.

Explore

Copy these diagrams.

$\frac{\square}{\square} + \frac{\square}{\square} =$
greatest sum

$\frac{\square}{\square} + \frac{\square}{\square} =$
least sum

Use the digits 1, 2, 4, and 8 to make the greatest sum and the least sum.
In each case, use each digit once.

Reflect & Share

Share your results with another pair of classmates.
Did you have the same answers?
If not, which is the greatest sum? The least sum?
What strategies did you use to add?

Connect

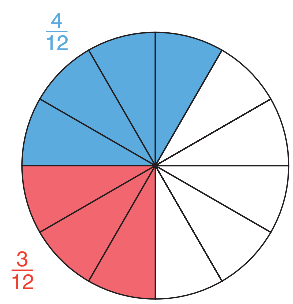
We can use equivalent fractions to add $\frac{1}{4} + \frac{1}{3}$.
Use equivalent fractions that have like denominators.
12 is a multiple of 3 and 4.
12 is a **common denominator**.

$\frac{1}{4} = \frac{3}{12}$ and $\frac{1}{3} = \frac{4}{12}$

So, $\frac{1}{4} + \frac{1}{3} = \frac{3}{12} + \frac{4}{12}$

$= \frac{7}{12}$

Both fractions are written with like denominators.



Look at the pattern in the equivalent fractions below.

$$\frac{1}{4} \xrightarrow{\times 3} \frac{3}{12} \qquad \frac{1}{4} = \frac{3}{12}$$

$$\frac{1}{3} \xrightarrow{\times 4} \frac{4}{12} \qquad \frac{1}{3} = \frac{4}{12}$$

So, to get an equivalent fraction, multiply the numerator and denominator by the same number.

We may also get equivalent fractions by dividing.

For example, $\frac{8}{10}$ can be written: $\frac{8 \div 2}{10 \div 2} = \frac{4}{5}$

$\frac{8}{10}$ and $\frac{4}{5}$ are equivalent fractions.

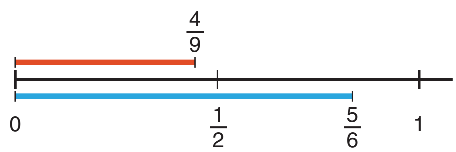
$\frac{4}{5}$ is in simplest form.

Example

Add: $\frac{4}{9} + \frac{5}{6}$

Estimate to check the sum is reasonable.

A Solution



$\frac{4}{9} + \frac{5}{6}$

Estimate first.

$\frac{4}{9}$ is about $\frac{1}{2}$.

$\frac{5}{6}$ is close to 1.

So, $\frac{4}{9} + \frac{5}{6}$ is about $1\frac{1}{2}$.

Use equivalent fractions to write the fractions with a common denominator.

List the multiples of 9: 9, **18**, 27, *36*, 45, …

List the multiples of 6: 6, 12, **18**, 24, 30, *36*, 42, …

18 is a multiple of 9 and 6, so 18 is a common denominator.

36 is also in both lists. So, 36 is another possible common denominator.

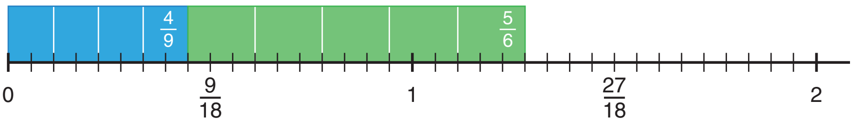
$$\frac{4}{9} \xrightarrow{\times 2} \frac{8}{18} \qquad \frac{4}{9} = \frac{8}{18}$$

$$\frac{5}{6} \xrightarrow{\times 3} \frac{15}{18} \qquad \frac{5}{6} = \frac{15}{18}$$

$$\frac{4}{9} + \frac{5}{6} = \frac{8}{18} + \frac{15}{18}$$
$$= \frac{23}{18}$$

Add the numerators.

We could have found this sum with fraction strips on a number line.



Since 23 > 18, this is an improper fraction.

To write the fraction as a mixed number:

$$\begin{aligned}\frac{23}{18} &= \frac{18}{18} + \frac{5}{18}\\ &= 1 + \frac{5}{18}\\ &= 1\tfrac{5}{18} \qquad \text{This is a mixed number.}\end{aligned}$$

The estimate was $1\frac{1}{2}$, so the answer is reasonable.

Recall that an improper fraction is a fraction with the numerator greater than the denominator.

Practice

Write all sums in simplest form.

Write improper fractions as mixed numbers.

1. Find a common denominator for each pair of fractions.

a) $\frac{1}{2}$ and $\frac{5}{8}$ b) $\frac{1}{8}$ and $\frac{2}{3}$ c) $\frac{2}{3}$ and $\frac{1}{9}$ d) $\frac{3}{5}$ and $\frac{2}{3}$

2. Copy and complete. Replace each $\square$ with a digit to make each equation true.

a) $\frac{3}{12} = \frac{\square}{4}$ b) $\frac{3}{4} = \frac{6}{\square}$ c) $\frac{3}{6} = \frac{\square}{4}$ d) $\frac{6}{8} = \frac{15}{\square}$

3. Add. Sketch a number line to model each sum.

a) $\frac{4}{9} + \frac{1}{3}$ b) $\frac{1}{2} + \frac{1}{3}$ c) $\frac{3}{8} + \frac{3}{2}$ d) $\frac{3}{4} + \frac{1}{6}$

4. Estimate, then add.

a) $\frac{3}{5} + \frac{4}{8}$ b) $\frac{1}{6} + \frac{5}{8}$ c) $\frac{5}{6} + \frac{7}{9}$

d) $\frac{3}{4} + \frac{4}{7}$ e) $\frac{1}{3} + \frac{2}{5}$ f) $\frac{1}{5} + \frac{5}{6}$

5. One page of a magazine had 2 advertisements.
One was $\frac{1}{8}$ of the page, the other $\frac{1}{16}$.
What fraction of the page was covered?
Show your work.

6. Which sum is greater? Show your thinking.
$\frac{2}{3} + \frac{5}{6}$ or $\frac{3}{4} + \frac{4}{5}$

7. **Assessment Focus** Three people shared a pie.
Which statement is true? Can both statements be true?
Use pictures to show your thinking.
a) Edna ate $\frac{1}{10}$, Farrah ate $\frac{3}{5}$, and Ferris ate $\frac{1}{2}$.
b) Edna ate $\frac{3}{10}$, Farrah ate $\frac{1}{5}$, and Ferris ate $\frac{1}{2}$.

8. Damara and Baldwin had to shovel snow to clear their driveway.
Damara shovelled about $\frac{3}{10}$ of the driveway.
Baldwin shovelled about $\frac{2}{3}$ of the driveway.
What fraction of the driveway was cleared of snow?

9. Each fraction below is written as the sum of two unit fractions.
Which sums are correct? Why do you think so?
a) $\frac{7}{10} = \frac{1}{5} + \frac{1}{2}$ b) $\frac{5}{12} = \frac{1}{3} + \frac{1}{4}$ c) $\frac{5}{6} = \frac{1}{3} + \frac{1}{3}$
d) $\frac{7}{12} = \frac{1}{2} + \frac{1}{6}$ e) $\frac{11}{18} = \frac{1}{2} + \frac{1}{9}$ f) $\frac{2}{15} = \frac{1}{10} + \frac{1}{30}$

A fraction with numerator 1 is a unit fraction.

10. **Take It Further** Add.
a) $\frac{3}{8} + \frac{1}{2} + \frac{3}{4}$ b) $\frac{1}{4} + \frac{3}{2} + \frac{2}{5}$ c) $\frac{2}{3} + \frac{5}{6} + \frac{4}{9}$

Reflect

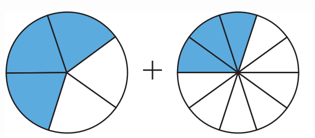
Suppose your friend has forgotten how to add two fractions with unlike denominators.
What would you do to help?

Mid-Unit Review

LESSON

Write all sums in simplest form.
Write improper fractions as mixed numbers.

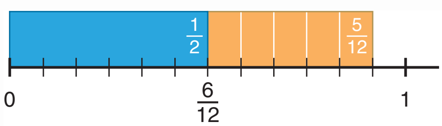
5.1 **1.** Use fraction circles. Model this picture, then find the sum.



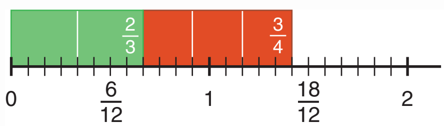
2. On Saturday, Howie hiked for $\frac{5}{12}$ h in the morning and $\frac{3}{6}$ h in the afternoon. What fraction of an hour did Howie spend hiking?

5.2 **3.** Write an addition equation for each picture.

a)



b)



4. Add. Sketch fraction strips and a number line to model each addition.

a) $\frac{2}{8} + \frac{3}{8}$ b) $\frac{2}{3} + \frac{1}{6}$

c) $\frac{3}{4} + \frac{2}{6}$ d) $\frac{1}{2} + \frac{2}{5}$

5. Find 3 different ways to add $\frac{2}{3} + \frac{5}{6}$. Draw pictures to help you explain each way.

5.3 **6.** Add. Estimate to check the sum is reasonable.

a) $\frac{4}{8} + \frac{5}{8}$ b) $\frac{1}{3} + \frac{3}{5}$

c) $\frac{1}{4} + \frac{1}{8}$ d) $\frac{5}{6} + \frac{7}{12}$

7. Takoda and Wesley are collecting shells on the beach in identical pails. Takoda estimates she has filled $\frac{7}{12}$ of her pail. Wesley estimates he has filled $\frac{4}{10}$ of his pail. Suppose the children combine their shells. Will one pail be full? Explain.

8. Each guest at Tai's birthday party brought one gift.
The circle graph shows the gifts Tai received.

a) What fraction of the gifts were:
   i) toys or books?
   ii) puzzles or toys?
   iii) games or puzzles?
   iv) books or games?

b) Which 2 types of gifts represent $\frac{1}{4}$ of all the gifts?
   Explain how you know.

5.4 Using Models to Subtract Fractions

Focus Use Pattern Blocks, fraction strips, and number lines to subtract fractions.

Explore

You will need congruent squares, grid paper, and coloured pencils.

Use these rules to create a rectangular design.
The design must be symmetrical.

- One-half of the squares must be red.
- One-third of the squares must be blue.
- The remaining squares must be green.

What fraction of the squares are green? How do you know?
How many squares did you use?
Explain why you used that number of squares.
Describe your design.
Record your design on grid paper.

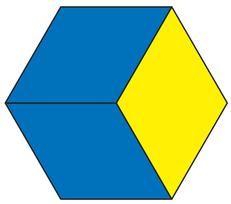
Reflect & Share

Compare your design with that of another pair of classmates.
If the designs are different, explain why your classmates' design obeys the rules.
How could you subtract fractions to find the fraction of the squares that are green?

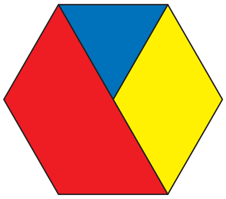
Connect

We can use models to subtract fractions.

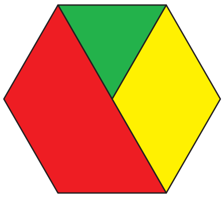
To subtract $\frac{2}{3} - \frac{1}{2}$, we can use Pattern Blocks.
The yellow hexagon represents 1. The blue rhombus represents $\frac{1}{3}$.
The red trapezoid represents $\frac{1}{2}$.
Place 2 blue rhombuses over the hexagon.



To subtract $\frac{1}{2}$, place a red trapezoid over the 2 blue rhombuses.



Find a Pattern Block equal to the difference.
The green triangle represents the difference.



The green triangle is $\frac{1}{6}$ of the hexagon.
So, $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

We can also use fraction strips and number lines to subtract.
To subtract fractions with unlike denominators, we use equivalent fractions.

Example

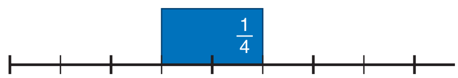
Subtract: $\frac{5}{8} - \frac{1}{4}$

A Solution

$\frac{5}{8} - \frac{1}{4}$

Think addition.
What do we add to $\frac{1}{4}$ to get $\frac{5}{8}$?
Use a number line that shows equivalent fractions for eighths and fourths. That is, use the eighths number line.
Place the $\frac{1}{4}$-strip on the eighths number line with its right end at $\frac{5}{8}$.

Equivalent fractions:
$\frac{1}{4} = \frac{2}{8}$



The left end of the strip is at $\frac{3}{8}$.
So, $\frac{5}{8} - \frac{1}{4} = \frac{3}{8}$

Practice

Use models.

1. Find equivalent fractions with like denominators for each pair of fractions.

a) $\frac{1}{2}$ and $\frac{5}{8}$ b) $\frac{1}{4}$ and $\frac{1}{3}$ c) $\frac{2}{3}$ and $\frac{1}{6}$ d) $\frac{3}{5}$ and $\frac{1}{2}$

2. Is each difference less than $\frac{1}{2}$ or greater than $\frac{1}{2}$?
How can you tell?

a) $\frac{5}{6} - \frac{1}{2}$ b) $\frac{7}{8} - \frac{1}{8}$ c) $\frac{4}{6} - \frac{1}{3}$ d) $1 - \frac{5}{6}$

3. Subtract. Sketch pictures to show each difference.

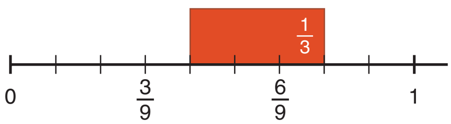
a) $\frac{3}{4} - \frac{2}{4}$ b) $\frac{4}{5} - \frac{1}{5}$ c) $\frac{2}{3} - \frac{1}{3}$ d) $\frac{5}{8} - \frac{3}{8}$

4. a) Write a rule you could use to subtract fractions with like denominators without using number lines or fraction strips.

b) Write 3 subtraction questions with like denominators.
Use your rule to subtract the fractions.
Use fraction strips and number lines to check your answers.

5. Write a subtraction equation for each picture.

a)



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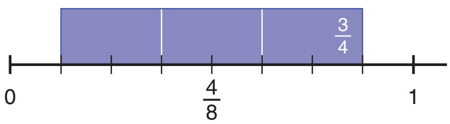
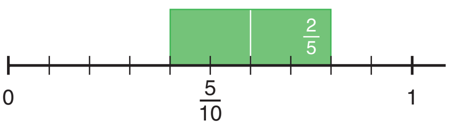
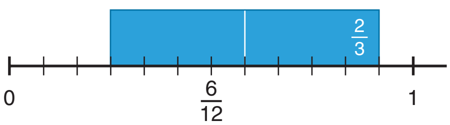


c)



d)



6. Subtract. Sketch pictures to show each difference.

a) $\frac{3}{8} - \frac{1}{4}$ b) $\frac{7}{10} - \frac{1}{2}$ c) $\frac{7}{8} - \frac{1}{2}$ d) $\frac{5}{6} - \frac{1}{4}$

7. Sergio has the lead role in the school play.
He still has to memorize $\frac{1}{2}$ of his lines.
Suppose Sergio memorizes $\frac{1}{3}$ of his lines today.
What fraction of his lines will he have left to memorize?
Show your work.

8. Freida has $\frac{3}{4}$ of a bottle of ginger ale.
She needs $\frac{1}{2}$ of a bottle of ginger ale for her fruit punch.
How much will be left in the bottle after Freida makes the punch?

9. A cookie recipe calls for $\frac{3}{4}$ cup of chocolate chips.
Spencer has $\frac{2}{3}$ cup. Does he have enough?
If your answer is yes, explain why it is enough.
If your answer is no, how much more does Spencer need?

10. Copy and replace each $\square$ with a digit, to make each equation true.
Try to do this more than one way.
a) $\frac{2}{3} - \frac{\square}{\square} = \frac{1}{3}$ b) $\frac{\square}{\square} - \frac{1}{5} = \frac{3}{5}$ c) $\frac{\square}{3} - \frac{2}{\square} = \frac{1}{6}$

11. **Assessment Focus** Kelly had $\frac{3}{4}$ of a tank of gas at the beginning of the week.
At the end of the week, Kelly had $\frac{1}{8}$ of a tank left.
a) Did Kelly use more or less than $\frac{1}{2}$ of a tank? Explain.
b) How much more or less than $\frac{1}{2}$ of a tank did Kelly use?
Show your work.

12. a) Which of these differences is greater than $\frac{1}{2}$?
Why do you think so?
i) $\frac{5}{6} - \frac{2}{3}$ ii) $\frac{5}{6} - \frac{1}{2}$ iii) $\frac{5}{6} - \frac{1}{6}$
b) Explain how you found your answers to part a.
Which other way can you find the fractions with a difference greater than $\frac{1}{2}$? Explain another strategy.

Reflect

When you subtract fractions with unlike denominators, how do you subtract?
Give 2 different examples.
Use diagrams to show your thinking.

5.5 Using Symbols to Subtract Fractions

Focus Use common denominators to subtract fractions.

Addition and subtraction are related operations.
You can use what you know about adding fractions to subtract them.

Explore

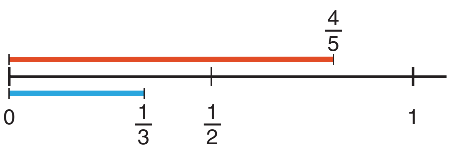
You will need fraction strips and number lines.
Find 2 fractions with a difference of $\frac{1}{2}$.
How many different pairs of fractions can you find?
Record each pair.

Reflect & Share

Discuss with your partner.
How are your strategies for subtracting fractions the same as your strategies for adding fractions? How are they different?
Work together to use common denominators to subtract two fractions.

Connect

To subtract $\frac{4}{5} - \frac{1}{3}$, estimate first.
$\frac{4}{5}$ is close to 1, and $\frac{1}{3}$ is about $\frac{1}{2}$.
So, $\frac{4}{5} - \frac{1}{3}$ is about $1 - \frac{1}{2} = \frac{1}{2}$.



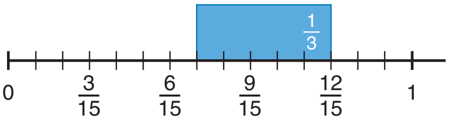
Use equivalent fractions to subtract.
Write $\frac{4}{5}$ and $\frac{1}{3}$ with a common denominator.
List the multiples of 5: 5, 10, **15**, 20, 25, …
List the multiples of 3: 3, 6, 9, 12, **15**, 18, …
15 is a multiple of 5 and 3, so 15 is a common denominator.

$$\frac{4}{5} \underset{\times 3}{\overset{\times 3}{=}} \frac{12}{15} \quad \text{and} \quad \frac{1}{3} \underset{\times 5}{\overset{\times 5}{=}} \frac{5}{15}$$

$$\begin{aligned} \frac{4}{5} - \frac{1}{3} &= \frac{12}{15} - \frac{5}{15} \\ &= \frac{7}{15} \end{aligned}$$

Think: 12 fifteenths minus 5 fifteenths is 7 fifteenths.

We could have used a fraction strip on a number line.



Example

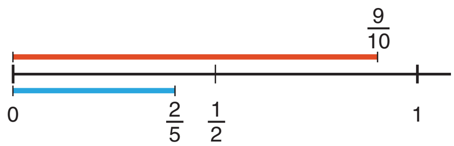
Subtract.

a) $\frac{9}{10} - \frac{2}{5}$ **b)** $\frac{5}{4} - \frac{1}{5}$

Estimate to check the answer is reasonable.

A Solution

a) $\frac{9}{10} - \frac{2}{5}$



Estimate.

$\frac{9}{10}$ is about 1. $\frac{2}{5}$ is close to $\frac{1}{2}$.

So, $\frac{9}{10} - \frac{2}{5}$ is about $1 - \frac{1}{2} = \frac{1}{2}$.

Since 10 is a multiple of 5,

use 10 as a common denominator.

$$\frac{2}{5} \xrightarrow{\times 2} \frac{4}{10}$$

$$\frac{2}{5} = \frac{4}{10}$$

$$\frac{9}{10} - \frac{2}{5} = \frac{9}{10} - \frac{4}{10}$$

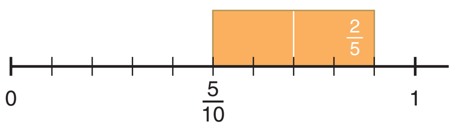
$$= \frac{5}{10}$$ This is not in simplest form.

$$= \frac{5 \div 5}{10 \div 5}$$ 5 is a factor of the numerator and denominator.

$$= \frac{1}{2}$$

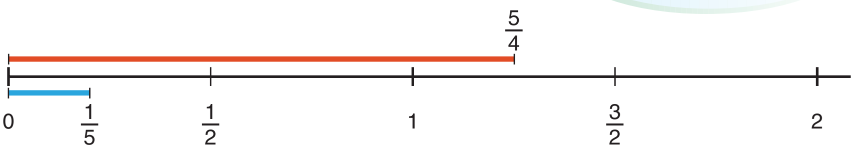
The estimate is $\frac{1}{2}$, so the difference is reasonable.

We could have used a fraction strip on a number line.



Another Strategy

b) $\frac{5}{4} - \frac{1}{5}$



Estimate.

$\frac{5}{4}$ is about 1. $\frac{1}{5}$ is close to 0.

So, $\frac{5}{4} - \frac{1}{5}$ is about $1 - 0 = 1$.

Find a common denominator.

List the multiples of 4: 4, 8, 12, 16, **20**, 24, …

List the multiples of 5: 5, 10, 15, **20**, 25, …

20 is a multiple of 4 and 5, so 20 is a common denominator.

$$\frac{5}{4} \overset{\times 5}{=} \frac{25}{20} \qquad\qquad \frac{1}{5} \overset{\times 4}{=} \frac{4}{20}$$

$$\frac{5}{4} - \frac{1}{5} = \frac{25}{20} - \frac{4}{20}$$

$$= \frac{21}{20}$$ This is an improper fraction.

$$\frac{21}{20} = \frac{20}{20} + \frac{1}{20}$$

$$= 1\frac{1}{20}$$

So, $\frac{5}{4} - \frac{1}{5} = 1\frac{1}{20}$

The estimate is 1, so the difference is reasonable.

Practice

Write all differences in simplest form.

1. Subtract.

a) $\frac{4}{5} - \frac{2}{5}$ b) $\frac{2}{3} - \frac{1}{3}$ c) $\frac{7}{9} - \frac{4}{9}$ d) $\frac{5}{7} - \frac{3}{7}$

2. Estimate, then subtract.

a) $\frac{2}{3} - \frac{1}{6}$ b) $\frac{5}{8} - \frac{1}{2}$ c) $\frac{3}{2} - \frac{7}{10}$ d) $\frac{11}{12} - \frac{5}{6}$

3. Subtract.

a) $\frac{3}{4} - \frac{2}{3}$ b) $\frac{4}{5} - \frac{2}{3}$ c) $\frac{7}{4} - \frac{4}{5}$ d) $\frac{3}{5} - \frac{1}{2}$

4. Subtract.

Estimate to check the answer is reasonable.

a) $\frac{4}{6} - \frac{1}{2}$ b) $\frac{5}{3} - \frac{3}{4}$ c) $\frac{7}{5} - \frac{5}{6}$ d) $\frac{5}{6} - \frac{3}{4}$

5. A recipe calls for $\frac{3}{4}$ cup of walnuts and $\frac{2}{3}$ cup of pecans.
Which type of nut is used more in the recipe?
How much more?

6. **Assessment Focus** On Saturday, Terri biked for $\frac{5}{6}$ h.

On Sunday, Terri increased the time she biked by $\frac{7}{12}$ h.

On Saturday, Bastien biked for $\frac{1}{2}$ h.

On Sunday, Bastien increased the time he biked by $\frac{3}{4}$ h.

a) Who biked longer on Sunday?
How can you tell?

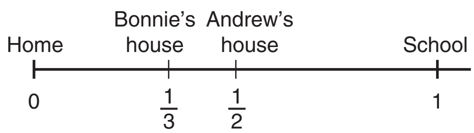
b) For how much longer did this person bike?

c) What did you need to know about fractions to answer these questions?

7. Write as many different subtraction questions as you can where the answer is $\frac{3}{4}$.
Show your work.

8. The difference of 2 fractions is $\frac{1}{2}$.
The lesser fraction is between 0 and $\frac{1}{4}$.
What do you know about the other fraction?

9. **Take It Further** Meagan walks from home to school at a constant speed.
It takes Meagan 3 min to walk the distance between Bonnie's house and Andrew's house.
How long does it take Meagan to get to school?



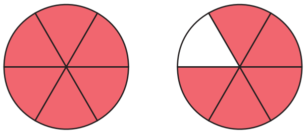
Reflect

Which fractions are easy to subtract?
Which are more difficult?
What makes them more difficult?
Give an example in each case.

5.6 Adding with Mixed Numbers

Focus Use models, pictures, and symbols to add with mixed numbers.

We have used fraction circles to model and add fractions.
We can also use fraction circles to model and add mixed numbers.
These fraction circles model $1\frac{5}{6}$.



1 whole $\quad \frac{5}{6}$

Explore

Use any materials you want.
A recipe calls for $1\frac{1}{3}$ cups of all-purpose flour and $\frac{5}{6}$ cup of whole-wheat flour.
How much flour is needed altogether?
How can you find out?
Show your work.

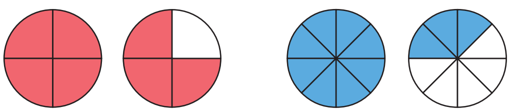
Reflect & Share

Describe your strategy.
Will your strategy work with all mixed numbers?
Test it with $2\frac{1}{3} + \frac{3}{4}$.
Use models or diagrams to justify your strategy.

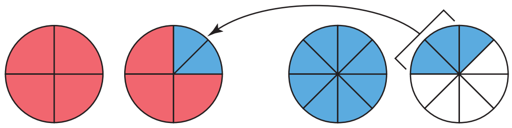
Connect

Use fraction circles to add: $1\frac{3}{4} + 1\frac{3}{8}$

Use fraction circles to model $1\frac{3}{4}$ and $1\frac{3}{8}$.



$1\frac{3}{4} \quad 1\frac{3}{8}$



Use eighths to fill the fraction circle for $\frac{3}{4}$.

1 whole and 1 whole and 1 whole and 1 eighth equals 3 wholes and 1 eighth.
So, $1\frac{3}{4} + 1\frac{3}{8} = 3\frac{1}{8}$

To add with mixed numbers, we can:

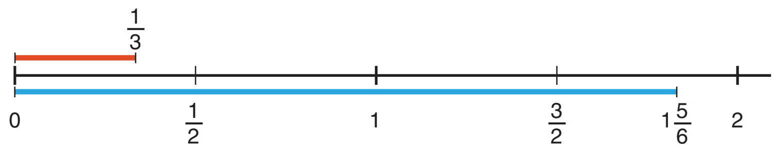
- Add the fractions and add the whole numbers separately. Or:
- Write each mixed number as an improper fraction, then add.

Example

Add: $\frac{1}{3} + 1\frac{5}{6}$

A Solution

$\frac{1}{3} + 1\frac{5}{6}$



Estimate:

$1\frac{5}{6}$ is close to 2.

So, $\frac{1}{3} + 1\frac{5}{6} > 2$, but less than $2\frac{1}{3}$

Add the fractions and
the whole number separately.

$\frac{1}{3} + 1\frac{5}{6} = \frac{1}{3} + \frac{5}{6} + 1$

Add the fractions: $\frac{1}{3} + \frac{5}{6}$

Since 6 is a multiple of 3,

use 6 as a common denominator.

$$\frac{1}{3} \overset{\times 2}{\underset{\times 2}{=}} \frac{2}{6}$$

$$\frac{1}{3} + \frac{5}{6} = \frac{2}{6} + \frac{5}{6}$$
$$= \frac{7}{6}$$

Since 7 > 6, this is an improper fraction.

To write the improper fraction as a mixed number:

$$\begin{aligned}\frac{7}{6} &= \frac{6}{6} + \frac{1}{6}\\ &= 1 + \frac{1}{6}\\ &= 1\frac{1}{6}\end{aligned}$$

$$\begin{aligned}\text{So, } \frac{1}{3} + \frac{5}{6} + 1 &= 1\frac{1}{6} + 1\\ &= 2\frac{1}{6}\end{aligned}$$

Then, $\frac{1}{3} + 1\frac{5}{6} = 2\frac{1}{6}$

This is close to the estimate of between 2 and $2\frac{1}{3}$, so the sum is reasonable.

Another Solution

Write the mixed number as an improper fraction, then add.

$$\begin{aligned}1\frac{5}{6} &= 1 + \frac{5}{6}\\ &= \frac{6}{6} + \frac{5}{6}\\ &= \frac{11}{6}\end{aligned}$$

Since 6 is a multiple of 3, use 6 as a common denominator.

$$\overset{\times 2}{\underset{\times 2}{\frac{1}{3} = \frac{2}{6}}}$$

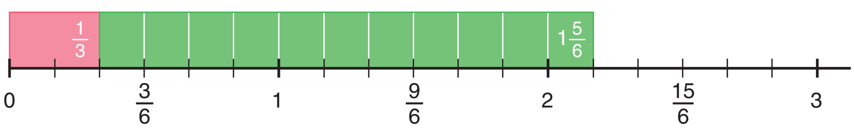
$$\begin{aligned}\frac{1}{3} + 1\frac{5}{6} &= \frac{2}{6} + \frac{11}{6}\\ &= \frac{13}{6}\end{aligned}$$

To write the fraction as a mixed number:

$$\begin{aligned}\frac{13}{6} &= \frac{12}{6} + \frac{1}{6}\\ &= 2 + \frac{1}{6}\\ &= 2\frac{1}{6}\end{aligned}$$

So, $\frac{1}{3} + 1\frac{5}{6} = 2\frac{1}{6}$

We can model this with a fraction strip on a number line.



Practice

Write all sums in simplest form.

1. Write each mixed number as an improper fraction in simplest form.

a) $1\frac{3}{6}$　b) $4\frac{2}{8}$　c) $1\frac{3}{4}$　d) $3\frac{3}{5}$

2. Write each improper fraction as a mixed number in simplest form.

a) $\frac{17}{5}$　b) $\frac{9}{4}$　c) $\frac{18}{4}$　d) $\frac{28}{6}$

3. Use Pattern Blocks to find each sum.

a) $1\frac{1}{6} + \frac{2}{6}$　b) $1\frac{2}{3} + \frac{2}{3}$　c) $1\frac{4}{6} + 2\frac{1}{2}$　d) $2\frac{1}{3} + 3\frac{5}{6}$

4. Find each sum.

a) $3\frac{2}{3} + 2\frac{1}{3}$　b) $1\frac{1}{8} + 3\frac{5}{8}$　c) $4\frac{2}{9} + 3\frac{5}{9}$　d) $2\frac{3}{5} + 5\frac{4}{5}$

5. Use fraction circles to find each sum.

a) $2\frac{5}{8} + \frac{3}{4}$　b) $2\frac{5}{12} + \frac{2}{3}$　c) $1\frac{3}{8} + 3\frac{3}{4}$　d) $2\frac{2}{5} + 1\frac{7}{10}$

6. We know $\frac{1}{2} + \frac{1}{5} = \frac{7}{10}$.
Use this result to find each sum.
Estimate to check the sum is reasonable.

a) $3\frac{1}{2} + \frac{1}{5}$　b) $\frac{1}{2} + 2\frac{1}{5}$　c) $3\frac{1}{2} + 2\frac{1}{5}$　d) $4\frac{1}{2} + 3\frac{1}{5}$

7. For each pair of numbers, find a common denominator. Then add.

a) $3\frac{1}{3} + \frac{1}{4}$　b) $\frac{1}{2} + 1\frac{9}{10}$　c) $\frac{3}{4} + 2\frac{3}{5}$　d) $\frac{3}{7} + 2\frac{1}{2}$

e) $4\frac{7}{8} + 1\frac{2}{3}$　f) $2\frac{3}{5} + 2\frac{2}{3}$　g) $5\frac{2}{5} + 1\frac{7}{8}$　h) $3\frac{5}{6} + 2\frac{1}{4}$

8. Two students, Galen and Mai, worked on a project.
Galen worked for $3\frac{2}{3}$ h.
Mai worked for $2\frac{4}{5}$ h.
What was the total time spent on the project?

9. **Assessment Focus** Joseph used $1\frac{3}{8}$ cans of paint to paint his room. Juntia used $2\frac{1}{4}$ cans to paint her room.

a) Estimate how many cans of paint were used in all.
b) Calculate how many cans of paint were used.
c) Draw a diagram to model your calculations in part b.

10. A recipe for punch calls for $2\frac{2}{3}$ cups of fruit concentrate and $6\frac{3}{4}$ cups of water.
How many cups of punch will the recipe make?
Show your work.

11. Use the fractions $1\frac{3}{5}$ and $2\frac{1}{10}$.

a) Add the fractions and the whole numbers separately.
b) Write each mixed number as an improper fraction.
c) Add the improper fractions.
d) Which method was easier: adding the mixed numbers or adding the improper fractions? Why do you think so?
When would you use each method?

12. An auto mechanic completed 2 jobs before lunch.
The jobs took $2\frac{2}{3}$ h and $1\frac{3}{4}$ h.
How many hours did it take the mechanic to complete the 2 jobs?

13. **Take It Further** Replace the $\square$ with an improper fraction or mixed number to make this equation true.
$3\frac{3}{5} + \square = 5$
Find as many answers as you can.
Draw diagrams to represent your thinking.

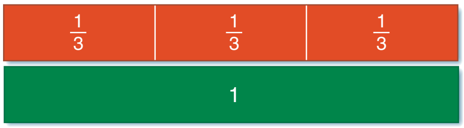
Reflect

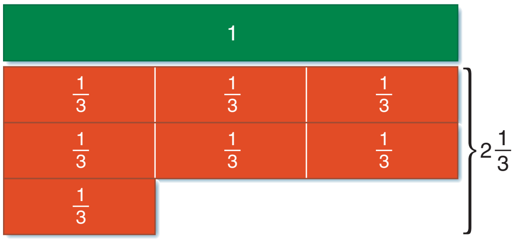
How is adding a mixed number and a fraction like adding two fractions?
How is it different?
Use examples to explain.

5.7 Subtracting with Mixed Numbers

Focus Use models, pictures, and symbols to subtract with mixed numbers.

We can use Cuisenaire rods to model fractions and mixed numbers.
Suppose the dark green rod is 1 whole, then the red rod is $\frac{1}{3}$.
So, seven red rods is $\frac{7}{3}$, or $2\frac{1}{3}$.





Explore

Use any materials you want.
A bicycle shop closed for lunch for $1\frac{2}{3}$ h on Monday and for $\frac{3}{4}$ h on Tuesday.
How much longer was the shop closed for lunch on Monday than on Tuesday?
How can you find out? Show your work.

Reflect & Share

Describe your strategy.
Will your strategy work with all mixed numbers?
Test it with $2\frac{1}{4} - \frac{3}{8}$.
Use models or diagrams to justify your strategy.

Connect

Use Cuisenaire rods to subtract: $1\frac{1}{2} - \frac{3}{4}$

Use Cuisenaire rods to model $1\frac{1}{2}$ and $\frac{3}{4}$.

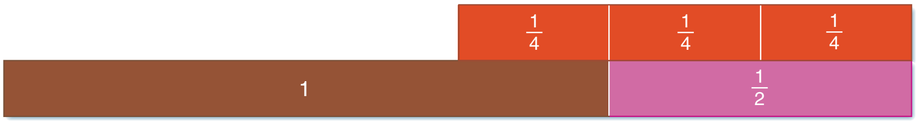
Let the brown rod represent 1 whole.

Then, the purple rod represents $\frac{1}{2}$ and the red rod represents $\frac{1}{4}$.

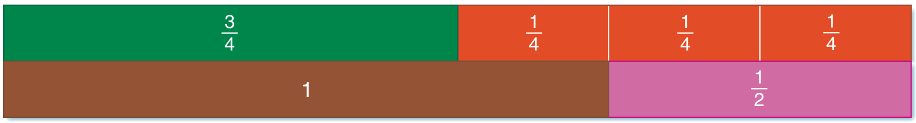
Model $1\frac{1}{2}$ with Cuisenaire rods.

Model $\frac{3}{4}$ with Cuisenaire rods.

Place the rods for $\frac{3}{4}$ above the rods for $1\frac{1}{2}$, so they align at the right.



Find a rod equal to the difference in their lengths.
The difference is equal to the dark green rod.



The dark green rod represents $\frac{3}{4}$ of the brown rod.
So, $1\frac{1}{2} - \frac{3}{4} = \frac{3}{4}$
To subtract with mixed numbers, we can:

- Subtract the fractions and subtract the whole numbers separately. Or:
- Write each mixed number as an improper fraction, then subtract.

Example

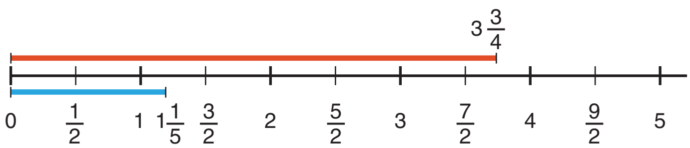
Subtract.

a) $3\frac{3}{4} - 1\frac{1}{5}$ b) $3\frac{1}{5} - \frac{3}{4}$

Estimate to check the answer is reasonable.

A Solution

a) $3\frac{3}{4} - 1\frac{1}{5}$



Estimate.

$3\frac{3}{4}$ is about 4. $1\frac{1}{5}$ is about 1.

So, $3\frac{3}{4} - 1\frac{1}{5}$ is between 2 and 3.

Subtract the fractions first: $\frac{3}{4} - \frac{1}{5}$

The denominators 4 and 5 have no common factors.

So, a common denominator is: $4 \times 5 = 20$.

$$\frac{3}{4} \overset{\times 5}{\underset{\times 5}{=}} \frac{15}{20} \qquad \frac{1}{5} \overset{\times 4}{\underset{\times 4}{=}} \frac{4}{20}$$

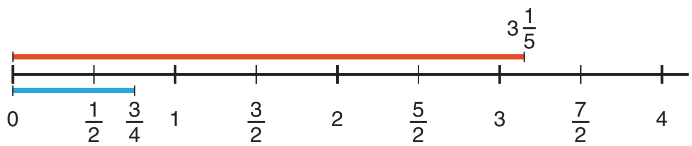
$$\frac{3}{4} - \frac{1}{5} = \frac{15}{20} - \frac{4}{20}$$
$$= \frac{11}{20}$$

Subtract the whole numbers: $3 - 1 = 2$

Then, $3\frac{3}{4} - 1\frac{1}{5} = 2\frac{11}{20}$

This is close to the estimate of between 2 and 3,
so the answer is reasonable.

b) $3\frac{1}{5} - \frac{3}{4}$



Estimate.

$3\frac{1}{5}$ is about 3.

$\frac{3}{4}$ is close to 1.

So, $3\frac{1}{5} - \frac{3}{4}$ is about $3 - 1 = 2$.

We cannot subtract the fractions because $\frac{1}{5} < \frac{3}{4}$.

So, write $3\frac{1}{5}$ as an improper fraction.

$$3\frac{1}{5} = 3 + \frac{1}{5}$$
$$= \frac{15}{5} + \frac{1}{5}$$
$$= \frac{16}{5}$$

Another Strategy
We could use fraction circles to subtract.

The denominators have no common factors.

So, a common denominator is: $4 \times 5 = 20$

$$\frac{16}{5} \xrightarrow[\times 4]{\times 4} \frac{64}{20} \qquad \frac{16}{5} = \frac{64}{20}$$

$$\frac{3}{4} \xrightarrow[\times 5]{\times 5} \frac{15}{20} \qquad \frac{3}{4} = \frac{15}{20}$$

$$\frac{16}{5} - \frac{3}{4} = \frac{64}{20} - \frac{15}{20}$$
$$= \frac{49}{20}$$
$$= \frac{40}{20} + \frac{9}{20}$$
$$= 2 + \frac{9}{20}$$
$$= 2\frac{9}{20}$$

So, $3\frac{1}{5} - \frac{3}{4} = 2\frac{9}{20}$

This is close to the estimate of 2, so the answer is reasonable.

Before we subtract the fraction parts of two mixed numbers, we must check the fractions to see which is greater. When the second fraction is greater than the first fraction, we cannot subtract directly.

Practice

Write all differences in simplest form.

1. Subtract.

a) $2\frac{3}{5} - 1\frac{2}{5}$ b) $3\frac{7}{8} - 1\frac{5}{8}$ c) $\frac{15}{4} - \frac{3}{4}$ d) $\frac{11}{6} - \frac{1}{6}$

2. Subtract. Use Cuisenaire rods. Sketch diagrams to record your work.

a) $1\frac{2}{3} - \frac{2}{6}$ b) $3\frac{1}{2} - 1\frac{2}{4}$ c) $3\frac{3}{10} - 2\frac{4}{5}$ d) $2\frac{1}{4} - \frac{1}{2}$

3. We know that $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$.
Use this result to find each difference.
Estimate to check the answer is reasonable.

a) $2\frac{2}{3} - \frac{1}{2}$ b) $2\frac{2}{3} - 1\frac{1}{2}$ c) $4\frac{2}{3} - 2\frac{1}{2}$ d) $5\frac{2}{3} - 1\frac{1}{2}$

4. Estimate, then subtract.

a) $\frac{7}{2} - \frac{5}{4}$ b) $\frac{13}{6} - \frac{8}{12}$ c) $\frac{5}{4} - \frac{3}{5}$ d) $\frac{9}{5} - \frac{1}{2}$

5. a) Subtract.

i) $3 - \frac{4}{5}$ ii) $4 - \frac{3}{7}$ iii) $5 - \frac{5}{6}$ iv) $6 - \frac{4}{9}$

b) Which methods did you use in part a?
Explain your choice.

6. For the fractions in each pair of numbers, find a common denominator.
Then subtract.

a) $3\frac{3}{4} - 1\frac{1}{5}$ b) $4\frac{9}{10} - 3\frac{1}{2}$ c) $3\frac{3}{4} - 1\frac{1}{3}$ d) $4\frac{5}{7} - 2\frac{2}{3}$

7. For each pair of mixed numbers below:

a) Subtract the fractions and subtract the whole numbers separately.
b) Write the mixed numbers as improper fractions, then subtract.
c) Which method was easier? Why do you think so?

i) $3\frac{3}{5} - 1\frac{3}{10}$ ii) $3\frac{3}{10} - 1\frac{3}{5}$

8. A flask contains $2\frac{1}{2}$ cups of juice.
Ping drinks $\frac{3}{8}$ cup of juice, then Preston drinks $\frac{7}{10}$ cup of juice.
How much juice is in the flask now? Show your work.

9. The running time of a movie is $2\frac{1}{6}$ h.
In the theatre, Jason looks at his watch and sees that $1\frac{1}{4}$ h has passed.
How much longer will the movie run?

10. Subtract.
a) $3\frac{2}{3} - 2\frac{7}{8}$
b) $5\frac{1}{2} - 3\frac{7}{9}$
c) $4\frac{3}{5} - 1\frac{2}{3}$
d) $4\frac{2}{5} - 1\frac{7}{8}$

11. **Assessment Focus** The students in two Grade 7 classes made sandwiches for parents' night.
Mr. Crowe's class used $5\frac{1}{8}$ loaves of bread.
Mme. Boudreau's class used $3\frac{2}{3}$ loaves of bread.
a) Estimate how many more loaves Mr. Crowe's class used.
b) Calculate how many more loaves Mr. Crowe's class used.
c) Draw a diagram to model your calculations in part b.
d) The two classes purchased 10 loaves.
How many loaves were left?

12. **Take It Further** Replace the □ with an improper fraction or mixed number to make this equation true.
$4\frac{1}{8} - \square = 1\frac{1}{2}$
Find as many answers as you can.
Draw diagrams to represent your thinking.

Reflect

You have learned to use improper fractions to subtract mixed numbers.
When is this not the better method? Use an example to explain.

Advertising Sales Representative

Magazines and newspapers make money by selling advertising space.

The advertising sales representative contacts companies whose products might be of interest to readers. She offers to sell them various sizes of advertisement space at different rates.
When talking about ads smaller than a full page, the sales rep uses fractions to describe them. It's much simpler to talk about a $\frac{2}{3}$-page ad instead of a 0.666 667 page ad!

The sales rep tries to sell combinations of ads that can fill pages, with no space left over. A sales rep has sold two $\frac{1}{4}$-page ads and one $\frac{1}{6}$-page ad. She wants to know the possible combinations of ad sizes she can sell to fill the rest of the page.
What might they be?

Writing a Complete Solution

A question often says "Show your work."
What does this mean?

When you are asked to show your work, you should show your thinking by writing a complete solution.

Work with a partner.
Compare these solutions.

Solution 1

A cookie recipe calls for $\frac{3}{8}$ cup of brown sugar and $\frac{1}{3}$ cup of white sugar.
Which type of sugar is used more in the recipe?
How much more?

Solution

Which is greater, $\frac{3}{8}$ or $\frac{1}{3}$?

Write $\frac{3}{8}$ and $\frac{1}{3}$ with a common denominator.
List the multiples of 8: 8, 16, 24, 32, ...
List the multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, ...

A common denominator is 24.

$\frac{3}{8} \overset{\times 3}{\underset{\times 3}{=}} \frac{9}{24}$ $\qquad$ $\frac{1}{3} \overset{\times 8}{\underset{\times 8}{=}} \frac{8}{24}$

$\frac{9}{24} > \frac{8}{24}$

So, $\frac{3}{8} > \frac{1}{3}$

Brown sugar is used more.

Subtract to find out how much more.

$\frac{9}{24} - \frac{8}{24} = \frac{1}{24}$

The recipe calls for $\frac{1}{24}$ cup more brown sugar.

Solution 2

$\frac{3}{8} = \frac{9}{24}$ $\qquad$ $\frac{1}{3} = \frac{8}{24}$

Answer: $\frac{9}{24}$, $\frac{1}{24}$

- Which solution is complete?
- Suppose this question is on a test. It is worth 4 marks.
  How many marks would you give each solution above?
  Justify your answers.

Make a list of things that should be included in a complete solution.

Reading and Writing in Math

Tips for writing a complete solution:

- Write down the question.
- Show all steps so that someone else can follow your thinking.
- Include graphs or pictures to help explain your thinking.
- Check that your calculations are accurate.
- Use math symbols correctly.
- Write a concluding sentence that answers the question.

Name: ______________ Date: __________

1. Which fraction is greater? How do you know?

$\frac{3}{5}, \frac{2}{3}$

2. Add.

$\frac{3}{4} + \frac{1}{12}$

3. Subtract.

$4\frac{5}{6} - \frac{17}{18}$

4. Marty drank $\frac{4}{5}$ cup of orange juice.

Kobe drank $\frac{3}{4}$ cup of orange juice.

a) Who drank more orange juice?
b) How much more orange juice did he drink?

Unit Review

What Do I Need to Know?

Adding and Subtracting Fractions

- ✓ Use models, such as Pattern Blocks, fraction circles, fraction strips, and number lines.
- ✓ Like denominators: add or subtract the numerators.
  For example, $\frac{5}{6} + \frac{2}{6} = \frac{7}{6}$ $\quad \frac{5}{6} - \frac{2}{6} = \frac{3}{6}$, or $\frac{1}{2}$
- ✓ Unlike denominators: Use a common denominator to write equivalent fractions, then add or subtract the numerators.
  For example:

$$\begin{aligned}&\frac{3}{4} + \frac{3}{5}\\ &= \frac{15}{20} + \frac{12}{20}\\ &= \frac{27}{20}, \text{ or } 1\frac{7}{20}\end{aligned}$$

$$\begin{aligned}&\frac{3}{4} - \frac{3}{5}\\ &= \frac{15}{20} - \frac{12}{20}\\ &= \frac{3}{20}\end{aligned}$$

Adding and Subtracting with Mixed Numbers

- ✓ Use models, such as fraction circles, Pattern Blocks, and Cuisenaire rods.
- ✓ Add or subtract the fractions and the whole numbers separately.
  For example:

$$\begin{aligned}&3\frac{5}{8} + 2\frac{1}{4}\\ &= 3 + 2 + \frac{5}{8} + \frac{1}{4}\\ &= 5 + \frac{5}{8} + \frac{2}{8}\\ &= 5 + \frac{7}{8}\\ &= 5\frac{7}{8}\end{aligned}$$

$$\begin{aligned}&3\frac{2}{3} - 1\frac{3}{5}\\ &= 3 - 1 + \frac{2}{3} - \frac{3}{5}\\ &= 2 + \frac{10}{15} - \frac{9}{15}\\ &= 2 + \frac{1}{15}\\ &= 2\frac{1}{15}\end{aligned}$$

Check that $\frac{2}{3} > \frac{3}{5}$.

- ✓ Write each mixed number as an improper fraction, then add or subtract.
  For example:

$$\begin{aligned}&1\frac{5}{6} + 1\frac{2}{5}\\ &= \frac{11}{6} + \frac{7}{5}\\ &= \frac{55}{30} + \frac{42}{30}\\ &= \frac{97}{30}, \text{ or } 3\frac{7}{30}\end{aligned}$$

$$\begin{aligned}&2\frac{1}{4} - 1\frac{1}{2}\\ &= \frac{9}{4} - \frac{3}{2}\\ &= \frac{9}{4} - \frac{6}{4}\\ &= \frac{3}{4}\end{aligned}$$

Since $\frac{1}{4} < \frac{1}{2}$, use improper fractions.

What Should I Be Able to Do?

LESSON

5.1 **1.** Add.
Use fraction circles.
Draw a picture to show each sum.

a) $\frac{8}{12} + \frac{5}{12}$ b) $\frac{3}{4} + \frac{2}{8}$

c) $\frac{1}{4} + \frac{2}{3}$ d) $\frac{1}{10} + \frac{3}{5}$

5.2 **2.** Add. Use fraction strips on number lines.
Draw a picture to show each sum.

a) $\frac{5}{9} + \frac{2}{3}$ b) $\frac{2}{3} + \frac{5}{6}$

c) $\frac{1}{6} + \frac{7}{12}$ d) $\frac{3}{8} + \frac{6}{8}$

3. Find 2 fractions that add to $\frac{5}{8}$.
Find as many pairs of fractions as you can.

5.3 **4.** Find a common denominator for each set of fractions.
Write equivalent fractions for each pair.

a) $\frac{3}{5}$ and $\frac{3}{4}$ b) $\frac{2}{5}$ and $\frac{3}{15}$

c) $\frac{4}{9}$ and $\frac{1}{2}$ d) $\frac{5}{8}$ and $\frac{1}{6}$

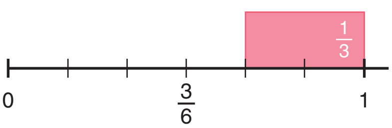
5. Add.

a) $\frac{1}{5} + \frac{3}{5}$ b) $\frac{1}{2} + \frac{3}{7}$

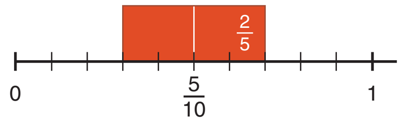
c) $\frac{2}{3} + \frac{3}{10}$ d) $\frac{3}{5} + \frac{1}{4}$

5.4 **6.** Write a subtraction equation for each picture.

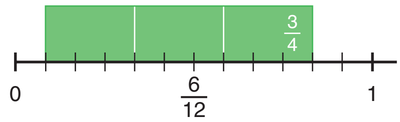
a)



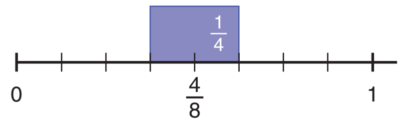
b)



c)



d)



7. Subtract. Draw a picture to show each difference.

a) $\frac{4}{5} - \frac{1}{5}$

b) $\frac{5}{6} - \frac{1}{3}$

c) $\frac{11}{12} - \frac{1}{2}$

8. Joyce and Javon each have the same MP3 player.
Joyce has used $\frac{7}{9}$ of her storage capacity.
Javon has used $\frac{5}{6}$ of his storage capacity.

a) Who has used more storage capacity?

b) How much more storage capacity has he or she used?

Show your work.

5.5 **9.** Subtract.

a) $\frac{9}{10} - \frac{2}{5}$ b) $\frac{7}{3} - \frac{5}{6}$

c) $\frac{8}{5} - \frac{1}{4}$ d) $\frac{9}{4} - \frac{2}{3}$

LESSON

10. Write a subtraction question that has each fraction below as the answer.
The two fractions that are subtracted should have unlike denominators.
a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{1}{10}$
d) $\frac{1}{6}$ e) $\frac{1}{4}$

11. Anton drank $\frac{3}{4}$ bottle of water.
Brad drank $\frac{7}{8}$ bottle of water.
a) Who drank more water?
b) How much more water did he drink?

12. The gas tank in Eddie's car is $\frac{5}{8}$ full. He uses $\frac{1}{4}$ tank of gas to run his errands. What fraction of a tank of gas is left?

5.6 13. Use fraction circles to find each sum.
a) $6\frac{1}{3} + \frac{1}{3}$ b) $1\frac{5}{12} + \frac{1}{6}$
c) $2\frac{3}{10} + 3\frac{1}{5}$ d) $5\frac{1}{4} + 1\frac{2}{5}$

14. Add.
a) $3\frac{5}{6} + \frac{4}{6}$ b) $4\frac{3}{8} + \frac{1}{4}$
c) $7\frac{3}{10} + 2\frac{4}{5}$ d) $2\frac{5}{9} + 5\frac{2}{3}$

15. Danielle mows lawns as a part-time job. On Monday, Danielle spent $1\frac{3}{4}$ h mowing lawns.
On Wednesday, she spent $1\frac{7}{8}$ h mowing lawns.
How much time did she spend mowing lawns over the 2 days?

5.7 16. Subtract. Draw a picture to show each difference.
a) $4\frac{1}{2} - \frac{3}{8}$ b) $3\frac{4}{9} - \frac{2}{3}$
c) $5\frac{5}{12} - 3\frac{5}{6}$ d) $4\frac{5}{8} - 2\frac{2}{3}$

17. Amelie wants to bake two kinds of muffins. One recipe calls for $1\frac{3}{4}$ cups of bananas.
The other recipe calls for $1\frac{7}{8}$ cups of cranberries.
a) Which recipe uses more fruit?
b) How much more fruit does the recipe in part a use?

18. Add or subtract as indicated.
a) $2\frac{2}{3} + 1\frac{1}{2}$ b) $3\frac{1}{3} - 1\frac{7}{10}$
c) $2\frac{1}{6} + 4\frac{7}{8}$ d) $3\frac{1}{2} - 2\frac{3}{4}$

19. On a trip from Edmonton to Saskatoon, Carly drove for $2\frac{1}{2}$ h, stopped for gas and lunch, then drove for $2\frac{2}{3}$ h.
The total trip took 6 h. How long did Carly stop for gas and lunch?
Express your answer as a fraction of an hour.

Practice Test

1. Add or subtract.
Draw a picture to show each sum or difference.
Write each sum or difference in simplest form.

a) $\frac{7}{5} + \frac{3}{5}$ **b)** $\frac{13}{10} - \frac{2}{3}$ **c)** $\frac{11}{12} - \frac{8}{12}$ **d)** $\frac{4}{9} + \frac{7}{6}$

2. Find two fractions that have a sum of $\frac{3}{5}$.
a) The fractions have like denominators.
b) The fractions have unlike denominators.

3. Find two fractions that have a difference of $\frac{1}{4}$.
a) The fractions have like denominators.
b) The fractions have unlike denominators.

4. Add or subtract.
a) $6\frac{3}{8} + 2\frac{1}{5}$ **b)** $3\frac{1}{10} - 1\frac{4}{5}$

5. Lana does yard work.
The table shows the approximate time for each job.
For one Saturday, Lana has these jobs:
– mow 3 small lawns
– mow 1 large lawn
– mow lawn/tidy yard in 2 places
– plant annuals in 1 place
Lana needs travel time between jobs,
and a break for lunch.
Do you think she will be able to do all the jobs? Justify your answer.

Job	Time
Mow small lawn	$\frac{1}{2}$ h
Mow large lawn	$\frac{3}{4}$ h
Mow lawn/tidy yard	$1\frac{1}{2}$ h
Plant annuals	$2\frac{1}{2}$ h

6. Write each fraction as the sum of two different unit fractions.
a) $\frac{3}{4}$ **b)** $\frac{5}{8}$

7. A fraction is written on each side of two counters.
All the fractions are different.
The counters are flipped and the fractions are added.
Their possible sums are: $1, 1\frac{1}{4}, \frac{7}{12}, \frac{5}{6}$
Which fractions are written on the counters?
Explain how you found the fractions.

Unit Problem

Publishing a Book

The students at Anishwabe School are preparing a special book for the school's 100th anniversary.
They finance the book by selling advertising space to sponsors.
The students sold the following space:

Full page	$\frac{1}{2}$ page	$\frac{1}{3}$ page	$\frac{1}{4}$ page	$\frac{1}{6}$ page	$\frac{1}{8}$ page
1	1	1	3	4	5

All the advertisements are to fit at the back of the book.
Sam asks: "How many pages do we need for the advertisements?"
Ruth asks: "Will the advertisements fill the pages?"
Jiba asks: "Is there more than one way to arrange these advertisements?"
Can you think of other questions students might ask?

1. Find the total advertising space needed.

2. Sketch the pages to find how the advertisements can be placed. Use grid paper if it helps.

3. Compare your group's sketch with those of other groups.
When you made your sketch, what decisions did you make about the shape of each advertisement?
Did other groups make the same decisions?
If your answer is no, explain how another group made its decisions.

4. What are the fewest pages needed to display the advertisements?
Will there be room for any other advertisements?
How can you tell?

5. What else might students need to consider as they prepare the layout for the book?

Check List

Your work should show:

- ✓ all calculations in detail
- ✓ diagrams of the layout for the advertisements
- ✓ a clear explanation of how you prepared the layout
- ✓ a clear explanation of which students received the prizes, and how much more the third student needed to sell

To encourage students to sell advertisements, the organizing committee offered prizes to the 2 students who sold the most space.

Sandra, Roy, and Edward are the top sellers.
This table shows the advertising space each of these students sold.

	Full page	$\frac{1}{2}$ page	$\frac{1}{3}$ page	$\frac{1}{4}$ page	$\frac{1}{6}$ page	$\frac{1}{8}$ page
Sandra		1			1	2
Roy	1				1	
Edward			1	1	1	1

6. Which two students sold the most space?
Show how you know.

7. How much more space would the third-place student have to sell to receive first prize?
Second prize? Show your work.

Reflect on Your Learning

Look back at the goals under *What You'll Learn*.
How well do you think you have met these goals?